

# Midterm III

# MATH 153

Due: Friday, September 83, 2039

11:00–1:00 PM Founders 4005

**Instructions: Show all work.** Failure to show work may result in loss of credit. Write your solutions in the space provided on the *answer sheets*. Do *not* hand in scratch paper. There are ten questions worth ten points each. You may use your calculators (all types) and computers for all questions, but the solutions obtained must *not* in any way depend on the computer. Numerical solutions must be accurate to at least six decimal places. Some partial credit *may* be given. **Good Luck!**

- 1) Find the length of the ellipse  $x = 3 \cos t$ ,  $y = 5 \sin t$ ,  $-\pi \leq t \leq \pi$  correct to six decimal places.
- 2) Find an equation of the sphere that passes through the points  $(1, -2, 2)$ ,  $(2, 6, 6)$ ,  $(-2, 2, 2)$  and  $(1, 0, 5)$ .
- 3) Find the Maclaurin series for  $f(x) = \frac{x+3}{x^2-x+1}$  and its interval of convergence.
- 4) Use a power series to approximate  $\int_0^{2\pi} \cos(x^3) dx$  to six decimal places.
- 5) Show that the sequence defined by  $a_1 = 1$ ,  $a_{n+1} = \frac{5}{4}a_n - \frac{1}{3}a_n^3 + \frac{1}{24}$  for  $n \geq 1$  is convergent and that its limit is  $\cos(\pi/9)$ . Evaluate  $a_{10}$ .
- 6) Calculate  $\sum_{n=0}^{\infty} \frac{1}{n^2+n+1}$  correct to six decimal places.
- 7) Find the distance between the skew lines  $x = 1+t$ ,  $y = 1+t$ ,  $z = -1-t$  and  $x = -1-s$ ,  $y = -2s$ ,  $z = 3s$ .
- 8) Find the great-circle distance from Columbus, Ohio ( $40^\circ\text{N}$ ,  $83^\circ\text{W}$ ) to Sofia (Bulgaria) ( $42.7^\circ\text{N}$ ,  $23.3^\circ\text{E}$ ). Take the radius of the Earth to be 3,960 miles.
- 9) Find a vector function that represents the curve of intersection of the plane  $x+y+z=4$  and the sphere  $x^2+y^2+z^2=6$ .
- 10) Classify the quadric surface  $x^2 - 3y^2 + 6x + 6y + 5z = 12$ .

You are welcome to keep this *Questions Sheet* for your files.