

# Final Exam

Saturday, July 46, 2038

# MATH 153

11:00 – 1:00 Founders 4005

**Instructions: Show all work.** Failure to show work may result in loss of credit. Write your solutions in the space provided on the *answer sheets*. Do *not* hand in scratch paper. There are twelve questions. You may use your scientific calculators (all types) and a one-page list of relevant formulae. Some partial credit *may* be given. **Good Luck!**

- 1) Determine whether the series  $\sum_{n=0}^{\infty} n2^{-n}$  converges or diverges. If it converges, find its sum.
- 2) Determine whether the series  $\sum_{n=1}^{\infty} (-1)^n (\sqrt{4n+3} - 2\sqrt{n})$  is convergent or divergent.
- 3) Expand  $e^{-x^2}$  as a power series. Determine the interval of convergence.
- 4) Find the length of the curve  $r = \theta^2$ ,  $0 \leq \theta \leq 2\pi$ .
- 5) Approximate the function  $f(x) = \ln(2x - x^2)$  by a Taylor polynomial of degree two at  $a = 1$ . How accurate is that approximation when  $0.8 \leq x \leq 1.2$ ?
- 6) Find the area of the triangle with vertices  $(-2, 5, 2)$ ,  $(6, -2, 2)$ ,  $(2, -2, 7)$ , and a unit vector orthogonal to the plane through these points.
- 7) Find an equation for the circle that passes through the points  $(2, 0)$ ,  $(0, -5)$ ,  $(7, 1)$ . Sketch the circle.
- 8) Find the distance between the skew lines  $x = y = z$  and  $-x = y = z + 1$ .
- 9) Find a vector function that represents the curve of intersection of the spheres  $x^2 + y^2 + z^2 = 4$  and  $x^2 + y^2 + z^2 - 2x = 0$ .
- 10) Evaluate the integral  $\int_0^1 ((5 + 3t^2)\mathbf{i} + (1 + te^t)\mathbf{j} - \ln(1 + t)\mathbf{k}) dt$ .
- 11) Find the unit tangent and unit normal vectors  $T(t)$ ,  $N(t)$  and the curvature of the curve  $\mathbf{r}(t) = \langle 3t, t^2 - 1, t^2 - 4t \rangle$ .
- 12) Find the velocity, acceleration, and speed of a particle with position vector  $\mathbf{r}(t) = t^2\mathbf{i} + \sin t\mathbf{j} + \cos t\mathbf{k}$ .

**Points:** Question # $n$  is worth  $\frac{50 - 2 \cos(2n\pi/3)}{3}$  points.

You are welcome to keep this *Questions Sheet* for your files.