

**First Exam**

Name: \_\_\_\_\_

**Math 568**

January 23, 2008

11:30 AM–12:18 PM

Show your work. No calculators.

1. (20 points). Suppose

$$\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}. \quad \text{Calculate the following:}$$

(a)  $2\mathbf{u} - \mathbf{v} =$

(b)  $\mathbf{u} \cdot \mathbf{v} =$

(c) the length of  $\mathbf{v}$ ,  $\|\mathbf{v}\| =$

(d) the orthogonal projection of  $\mathbf{u}$  onto  $\mathbf{v}$ ,  $\text{proj}_{\mathbf{v}}(\mathbf{u}) =$

(e)  $\cos \theta$ , where  $\theta$  is the angle between  $\mathbf{u}$  and  $\mathbf{v}$ .

2. (18 points). Consider the line  $3x - y = 1$  in  $\mathbb{R}^2$ .

(a) Find a normal vector  $\mathbf{n}$  to the line.

(b) Find vectors  $\mathbf{p}$  and  $\mathbf{u}$  so that the line can be written in the form  $\mathbf{x} = \mathbf{p} + t\mathbf{u}$ , where  $\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$  and  $t$  varies over the real numbers.

(c) Find the distance from the closest point on the line to the origin.

3. (12 points).

(a) Put the matrix  $\begin{bmatrix} 1 & 3 & 8 \\ 1 & 3 & 5 \\ 2 & 6 & 9 \end{bmatrix}$  in row echelon form.

(b) What is its rank?

(c) Put the matrix  $\begin{bmatrix} 0 & 2 & 2 & 8 \\ 0 & 0 & 2 & 6 \\ 1 & 0 & -3 & 2 \end{bmatrix}$  in *reduced* row echelon form.

4. (8 points). Define the *span* of a set of vectors  $\{\mathbf{v}_1, \dots, \mathbf{v}_k\}$  in  $\mathbb{R}^n$ .

5. (8 points). Define what it means for a set of vectors  $\{\mathbf{v}_1, \dots, \mathbf{v}_k\}$  to be *linearly independent*.

6. (20 points). Suppose

$$\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

(a) In general, what system of linear equations do you need to solve in order to show that the vector  $\mathbf{b}$  is in the span of  $\{\mathbf{u}, \mathbf{v}\}$ ?

(b) Is the vector  $\mathbf{b} = \begin{bmatrix} 1 \\ 6 \\ 7 \end{bmatrix}$  in the span of  $\{\mathbf{u}, \mathbf{v}\}$ ? If not, why not?

If so, write it as a specific linear combination of  $\mathbf{u}$  and  $\mathbf{v}$ .

7. (14 points). Find a nontrivial linear combination of the vectors  $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ,  $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ , and  $\mathbf{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$  which equals the zero vector.

1 (20)	
2 (18)	
3 (12)	
4 ( 8)	
5 ( 8)	
6 (20)	
7 (14)	
Total	