First Exam **Name:**________________________________

Math 568

January 23, 2008

9:30–10:18 AM

Show your work. No calculators.

1. (20 points). Suppose

$$\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}. \quad \text{Calculate the following:}$$

(a) $3\mathbf{u} - \mathbf{v} =$

(b) $\mathbf{u} \cdot \mathbf{v} =$

(c) the length of $\mathbf{v}$, $\|\mathbf{v}\| =$

(d) the orthogonal projection of $\mathbf{u}$ onto $\mathbf{v}$, $\text{proj}_{\mathbf{v}}(\mathbf{u}) =$

(e) $\cos\theta$, where θ is the angle between $\mathbf{u}$ and $\mathbf{v}$.

2. (18 points). Consider the line $x + 2y = 1$ in $\mathbb{R}^2$.

(a) Find a normal vector $\mathbf{n}$ to the line.

(b) Find vectors $\mathbf{p}$ and $\mathbf{u}$ so that the line can be written in the form $\mathbf{x} = \mathbf{p} + t\mathbf{u}$, where $\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ and t varies over the real numbers.

(c) Find the distance from the closest point on the line to the origin.

3. (12 points).

(a) Put the matrix $\begin{bmatrix} 1 & 3 & 5 \\ 2 & 6 & 11 \\ 1 & 3 & 8 \end{bmatrix}$ in row echelon form.

(b) What is its rank?

(c) Put the matrix $\begin{bmatrix} 0 & 2 & 1 & 4 \\ 0 & 0 & 2 & 6 \\ 1 & 0 & -3 & 2 \end{bmatrix}$ in *reduced* row echelon form.

4. (8 points). Define the *span* of a set of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ in $\mathbb{R}^n$.

5. (8 points). Define what it means for a set of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ to be *linearly independent*.

6. (20 points). Suppose

$$\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

(a) In general, what system of linear equations do you need to solve in order to show that the vector $\mathbf{b}$ is in the span of $\{\mathbf{u}, \mathbf{v}\}$?

(b) Is the vector $\mathbf{b} = \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix}$ in the span of $\{\mathbf{u}, \mathbf{v}\}$? If not, why not?

If so, write it as a specific linear combination of $\mathbf{u}$ and $\mathbf{v}$.

7. (14 points). Find a nontrivial linear combination of the vectors $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ which equals the zero vector.

1 (20)	
2 (18)	
3 (12)	
4 (8)	
5 (8)	
6 (20)	
7 (14)	
Total	