Second Exam **Name:**_____
Math 568
February 25, 2008
11:30 AM–12:18 PM

Show your work. No calculators.

1. (12 points). Suppose

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 8 & 6 & 7 & 5 \\ 9 & 8 & 7 & 6 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 5 & 1 & 0 & 1 \\ 2 & 6 & -1 & 1 & 0 \\ 3 & 7 & 1 & 0 & 1 \\ 4 & 8 & -1 & 0 & 0 \end{bmatrix}.$$

   (a) What size is the matrix AB? (How many rows and how many columns?)

   (b) If A had one more row, would the product of A and B still be defined?

   (c) What entry does AB have in its second row and third column?

2. (5 points). Define what is meant by an *eigenvalue* a square matrix A.

3. (18 points).

$$A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

(a) $\det A =$

(b) $\det B =$

(c) $A^{-1} =$

(d) $B^{-1} =$

4. (10 points). Suppose V is a subspace of $\mathbb{R}^n$. Define what is meant by

   (a) a *basis* for V:

   (b) the *dimension* of V:

5. (5 points). Suppose A is an $m \times n$ matrix. Define what is meant by its *column space*, $\text{col}(A)$. (In particular, say what it is a subspace of.)

6. (12 points). The following matrices A and B are row equivalent, where

$$A = \begin{bmatrix} 3 & -1 & 7 & 3 & 9 \\ -2 & 2 & -2 & 7 & 5 \\ -5 & 9 & 3 & 3 & 4 \\ -2 & 6 & 6 & 3 & 7 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -1 & 7 & 0 & 6 \\ 0 & 2 & 4 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) What is a basis for col(B) (the column space of B)?

(b) What is a basis for col(A)?

(c) What is the dimension of col(A)?

(d) What is the dimension of the row space, row(A)?

7. (6 points). Suppose A is 15×9 (15 rows, 9 columns). If its null space, $\text{null}(A)$, has dimension 4, what are the possible values for the dimension of $\text{col}(A)$?

8. (8 points). Suppose

$$A = \begin{bmatrix} -1 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

Compute each of the following products or explain why it is not defined.

(a) $AB =$

(b) $BA =$

9. (12 points). The matrix

$$A := \begin{bmatrix} 4 & -3 & 6 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

has 1 as an eigenvalue. Find a basis for the corresponding eigenspace.

10. (12 points). A 9×9 matrix B has the property that $B^5 = \mathbf{0}$ (i.e., the fifth power of B is the 0-matrix). Prove that any eigenvalue of B is 0.

What can you say about $\det B$?

1 (12)	
2 (5)	
3 (18)	
4 (10)	
5 (5)	
6 (12)	
7 (6)	
8 (8)	
9 (12)	
10 (12)	
Total	