

Second Exam

Name: _____

Math 568

February 25, 2008

9:30–10:18 AM

Show your work. No calculators.

1. (12 points). Suppose

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 8 & 7 & 6 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 5 & 1 & 0 & 1 \\ 2 & 6 & -1 & 1 & 0 \\ 3 & 7 & 1 & 0 & 1 \\ 4 & 8 & -1 & 1 & 0 \end{bmatrix}.$$

- (a) What size is the matrix AB ? (How many rows and how many columns?)
- (b) If A had one more row, would the product of A and B still be defined?
- (c) What entry does AB have in its second row and third column?

2. (5 points). Define what is meant by an *eigenvalue* a square matrix A .

3. (18 points).

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) $\det A =$

(b) $\det B =$

(c) $A^{-1} =$

(d) $B^{-1} =$

4. (10 points). Suppose V is a subspace of \mathbb{R}^n . Define what is meant by

(a) a *basis* for V :

(b) the *dimension* of V :

5. (5 points). Suppose A is an $m \times n$ matrix. Define what is meant by its *column space*, $\text{col}(A)$. (In particular, say what it is a subspace of.)

6. (12 points). The following matrices A and B are row equivalent, where

$$A = \begin{bmatrix} 3 & -1 & 7 & 3 & 9 \\ -2 & 2 & -2 & 7 & 5 \\ -5 & 9 & 3 & 3 & 4 \\ -2 & 6 & 6 & 3 & 7 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -1 & 7 & 0 & 6 \\ 0 & 2 & 4 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

- (a) What is a basis for $\text{col}(B)$ (the column space of B)?

- (b) What is a basis for $\text{col}(A)$?

- (c) What is the dimension of $\text{col}(A)$?

- (d) What is the dimension of the row space, $\text{row}(A)$?

7. (6 points). Suppose A is 13×10 (13 rows, 10 columns). If its null space, $\text{null}(A)$, has dimension 4, what are the possible values for the dimension of $\text{col}(A)$?

8. (8 points). Suppose

$$A = [1 \quad -1 \quad 0], \quad B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Compute each of the following products or explain why it is not defined.

(a) $AB =$

(b) $BA =$

9. (12 points). The matrix

$$A := \begin{bmatrix} 3 & -3 & 6 \\ 0 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$$

has 2 as an eigenvalue. Find a basis for the corresponding eigenspace.

10. (12 points). A 10×10 matrix B has the property that $B^4 = \mathbf{0}$ (i.e., the fourth power of B is the 0-matrix). Prove that any eigenvalue of B is 0.

What can you say about $\det B$?

| | |
|---------|--|
| 1 (12) | |
| 2 (5) | |
| 3 (18) | |
| 4 (10) | |
| 5 (5) | |
| 6 (12) | |
| 7 (6) | |
| 8 (8) | |
| 9 (12) | |
| 10 (12) | |
| Total | |