A REVIEW OF KARL-GEORG STEFFENS' BOOK ON THE HISTORY OF APPROXIMATION THEORY

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NOTE. Versions of this review will appear in both SIAM Review (SR) and Journal of Approximation Theory (JAT).

If I say that Fejér was a student of Schwarz and that Fekete was a student of Fejér, then this insider joke will make every JAT person worth her salt laugh uncontrollably whereas, to outsiders, one needs to explain patiently that Fejér, which means *white* in Hungarian, was born Weiß, and that Fekete, which means *black* in Hungarian, was born Schwarz. Isn't this hilarious even if it's not fully accurate since Fekete was born with the name Fekete?[1] It could also be called a briefest history of approximation theory. The book under review is not as brief but no less entertaining albeit for reasons that are partially less than complimentary to the author.

My good friend, Zeev Ditzian, used to say in his heavily accented English that the international language of mathematics is broken English. Oh boy, was Zeev right. Anyone who has the slightest doubt should take a look at the book under review. I guarantee that it is an unlimited source of atrocious usage of the English language combined with equally dreadful stylistic and typesetting decisions or the lack thereof. It would keep a conscientious and competent Editor and/or copy-Editor busy for a long time. Since the people at Birkhäuser don't seem to be embarrassed to publish such a book, who am I to suggest that they should be.

In what follows, I will concentrate on the contents of the book, and I will no longer complain about the language and grammar used to convey the intentions of the author. In order to demonstrate my good will, I, hereby, donate him a few hundred commas and other punctuation marks which could be used to improve the second and subsequent editions of his book. In exchange, I would also want the author to remove at least half of the mysteriously placed line indentations. I will even throw in a bunch of dx's for free.

The stated goal of Karl-Georg Steffens' book is *to describe the early development of approximation theory... (until) 1919....*

Now let me pose a question to the Reader of this review. Suppose, I wrote a book on the life of Galileo and failed to mention that eventually even the Catholic Church accepted that *eppur si muove*, or, as a more mathematical example, suppose a book on Fermat would give no hint whatsoever how his theorem turned out to be, then would the Reader feel to have been cheated out of some tidbits or is it legitimate not to reveal to the Reader at least some of the essential events related to the subject of the book, whether or not they happened after the departure of the person under consideration? Let the Reader decide this dilemma.

I, personally, find it hard to read details about some investigations of, say, S. N. Bern-

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[1] I bet $10 that his father was born Schwarz. The first person disproving my conjecture should contact me to collect her prize.

stein on pp. 182–183 in Section 5.3.2 (and the same repeated on p. 187), and then have no reference whatsoever to important work done by Dick Varga and others; see, e.g., `http://mathworld.wolfram.com/BernsteinsConstant.html`, or google "bernstein constant". In conjunction with this, I find it rather amusing that (Joe) Stalin is mentioned in the book whereas (Herbert) Stahl is not.[2]

How about Bernstein–Jackson without Ditzian–Totik? No approximator could imagine such an omission, and the author missed a tremendous opportunity to educate the public about contemporary advances of classical approximation theory.

Well, the author anticipates my arguments in the preface where he writes *we set as an endpoint the year 1919*, but then he himself violates this rule more than once (but, unfortunately, not sufficiently many times).

Now it is time to make a confession. My review is based on what I read and I will not discuss those parts that I have not read.

The Good. I learned lots of new names, new stories, and new connections. I don't mean people such as M. G. Kreyn who I soon figured out is our good old approximator M. G. Krein, but, for instance, Antoni-Bonifatsi Pavlovich Psheborski (1879–1941) whose name drew a miserly 3 hits on Google, not counting multiplicity, and Konstantin Aleksandrovich Posse (1847–1928). I knew some results of the latter, I even used his inequalities on numerous occasions, but it was both educational and entertaining to read about his life. On the other hand, I couldn't figure out what the relationship of Ivan Ivanovich Ivanov was to approximation theory; see p. 196. In any case, he must have had some weird parents although he is not alone, Google has 1500+ hits for such names even when searching with quotation marks.

The Bad. The book is a rich source of incorrect mathematical statements. For instance, Theorem 3.4 on p. 103 is deduced from Theorem 3.3 on p. 102 by a totally false argument; see the paragraph starting with "since" after Theorem 3.3. By the way, z in the latter should be x. Another example is on p. 162 where the author is confusing algebraic and trigonometric polynomials (neither I_m nor J_m are algebraic polynomials). This is especially odd since in footnote 47 on p. 159 the author promises to stick to algebraic polynomials. What about the constant K in Theorem 5.2 on p. 180? I will let the Reader figure out what's wrong with the placement of K in the statement of the theorem. My advice is that the Reader should not even attempt to learn actual approximation theory from this book. Luckily, there is an abundance of excellent books on all levels for both pure and applied mathematicians, for non-mathematical scientists, and even for non-scientific type practitioners of mathematics.

The Ugly. The book was neither spell-checked nor proof-read. For instance, [1] is listed as *draft*, Bernstein's 3 page paper [Bern11] is referred to as a *monograph* on p. 164, there is a *for for* on the top of p. 159, and the innovative word *secretretary* appears on p. 98. These are not isolated examples. What the heck is *the 19th name* anyway; see the section titled *Explanations*, p. 201. Will someone please explain? What about *Jacobian polynomials*; see p. 57? Oops, I was wrong. Even Google gave 57 hits for that as opposed to 76,700 for the name the rest of us use. We also learn that *Markov strings* made A. A. Markov famous. Not according to Google (53 hits). If I had a say, I would vote for *Markov chains.* Google agrees with over 2 million hits.

By the way, I found some of the (misleading) information about George G. Lorentz

[2]If you are a JAT person and didn't get the joke, then shame on you.

on p. 178 quite amusing.[3]

It is equally amusing that the author uses the Christian symbol of a cross to indicate the death of Fejér; see footnote 35 on p. 150. In addition, as opposed to what is stated there, he was not called *Leopold* until 1900. The latter is simply the translation of his given name Lipót.

For the uninitiated, in the book, the symbol * means birth, whereas † means death, and "R.I.P." means, I guess, "rest in peace".

Speaking of Fejér (and Bernstein), the author could have at least mentioned the notion of positive and monotone operators which ended up occupying a central position in contemporary approximation theory.

In summary, both SR and JAT people will find plenty of old and new information in the book although the JAT people will be in a better position to decide the correctness of the information as presented in it. To be honest, sometimes separating the truth from the half-truth might be impossible, and it is unfair to blame the author for not giving us the truth and nothing but the truth. In particular, I would love to find out the truth and nothing but the truth about what happened between Bernstein and Jackson in 1911. Have they met or not? Have they discussed their results or not? Has Bernstein played any role in Jackson's direct theorems, and should Bernstein get more credit for the direct theorems? Similarly, what was Jackson's role, I mean, rôle (as the book prefers it) if any, in Bernstein's indirect theorems?

I missed the well-known story about Landau and a confirmation whether or not it was true. For the uninitiated, Landau told Jackson to prove the *Jackson* theorems but then, seeing how beautifully they turned out, he regretted not keeping the project to himself.

As far as I am concerned, the best parts of the book are the various quotations given in Russian (and their hilarious English translations). I especially liked those that demonstrate how even some of the brightest mathematicians became politically imbecilized during the Soviet Era; see, for instance, p. 71. As a counterexample, I learned that, according the S. N. Bernstein, dialectic materialism leads to mathematical illiteracy; see p. 177. Poor Sergey was fired for this. This was not the last time he was punished for not being an idiot.

In conclusion, Karl-Georg Steffens' book is necessary but far from sufficient for studying the history of approximation theory. Despite its numerous weaknesses, it does contain plenty of useful information that is not readily or not at all available elsewhere. Even if I had not been given a freebie review copy, I would have probably purchased it anyway, especially since the book is dedicated to Garald I. Natanson who had been my M.S. advisor so that the author and I are mathematical cousins so to speak.

Finally, what about the Reader who is now ready to study the history of approximation theory? What should she read and where should he start? I suggest to go to Allan Pinkus' and Carl de Boor's `http://www.math.technion.ac.il/hat/`[4] which has more than enough food for thought.[5]

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[3]Our beloved George died on 1/1/2006 at the age of almost 96 (the date is given using the American month/day/year notation).

[4]As I write this, Haifa is under attack.

[5]Incredibly, not a single paper ever published in JAT has the word "history" in its title.

Oops. I forgot to tell the Reader what approximation theory is. Let me present the promotional material, possibly written by the author, which, among others, can be found on the back cover of the book.

> The problem of approximating a given quantity is one of the oldest challenges faced by mathematicians. Its increasing importance in contemporary mathematics has created an entirely new area known as Approximation Theory. The modern theory was initially developed along two divergent schools of thought: the Eastern or Russian group, employing almost exclusively algebraic methods, was headed by Chebyshev together with his coterie at the Saint Petersburg Mathematical School, while the Western mathematicians, adopting a more analytical approach, included Weierstrass, Hilbert, Klein, and others.
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> This work traces the history of approximation theory from Leonhard Euler's cartographic investigations at the end of the 18th century to the early 20th century contributions of Sergei Bernstein in defining a new branch of function theory. One of the key strengths of this book is the narrative itself. The author combines a mathematical analysis of the subject with an engaging discussion of the differing philosophical underpinnings in approach as demonstrated by the various mathematicians. This exciting exposition integrates history, philosophy, and mathematics. While demonstrating excellent technical control of the underlying mathematics, the work is focused on essential results for the development of the theory.
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> The exposition begins with a history of the forerunners of modern approximation theory, i.e., Euler, Laplace, and Fourier. The treatment then shifts to Chebyshev, his overall philosophy of mathematics, and the Saint Petersburg Mathematical School, stressing in particular the roles played by Zolotarev and the Markov brothers. A philosophical dialectic then unfolds, contrasting East vs. West, detailing the work of Weierstrass as well as that of the Goettingen school led by Hilbert and Klein. The final chapter emphasizes the important work of the Russian Jewish mathematician Sergei Bernstein, whose constructive proof of the Weierstrass theorem and extension of Chebyshev's work serve to unify East and West in their approaches to approximation theory.
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> Appendices containing biographical data on numerous eminent mathematicians, explanations of Russian nomenclature and academic degrees, and an excellent index round out the presentation.

R.I.P.

REFERENCES

[1] Allan Pinkus, *Weierstrass and approximation theory*, J. Approx. Theory, **107** (2000), 1–66.