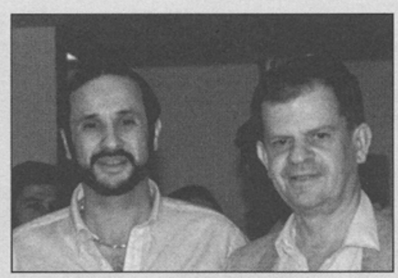
Gabor Szegő: 1895–1985

Richard Askey and Paul Nevai

The international mathematics community has recently celebrated the 100th anniversary of *Gabor Szegő*'s birth.[1]

Gabor Szegő was 90 years old when he died. He was born in Kunhegyes on January 20, 1895, and died in Palo Alto on August 7, 1985. His mother's and father's names were Hermina Neuman and Adolf Szegő, respectively. His birth was formally recorded at the registry of the Karcag Rabbinical district on January 27, 1895. He came from a small town of approximately 9 thousand inhabitants in Hungary (approximately 150 km southeast of Budapest), and died in a town in northern California, U.S.A., with a population of approximately 55 thousand, near Stanford University and just miles away from Silicon Valley. So many things happened during the 90 years of his life that shaped the politics, history, economy, and technology of our times that one should not be surprised that the course of Szegő's life did not follow the shortest geodesic curve between Kunhegyes and Palo Alto.

I (R. A.) first met Szegő in the 1950s when he returned to St. Louis to visit old friends, and I was an instructor at Washington University. Earlier, when I was an undergraduate there, I had used a result found by Hsien Yu Hsu in his Ph.D. thesis at Washington University under Szegő. This was in the first paper I wrote. While I was at the University of Chicago in the early 1960s, Szegő visited. I still remember seeing him at one end of the hall and a graduate student, Stephen Vági, at the other end of the same hall. They walked toward each other and both started to speak in Hungarian. I am certain they had not met before, and I have always wondered how Szegő recognized another former Hungarian. In 1972, I spent a month in Budapest and Szegő was there. We talked most days, and although his health was poor and his memory was not as good as it had been a few years earlier, we had some very useful discussions. Three years earlier, also in Budapest, Szegő



Paul Nevai and Richard Askey

Richard Askey went to Washington University, where he first encountered orthogonal polynomials and learned a little about Gabor Szegő's work. His Ph.D. at Princeton was on series of Jacobi polynomials, a very un-Princetonian topic at that time. Then he went from doing norm inequalities, to proving positivity results, to proving identities, getting more old-fashioned each time.

After having lived through the end of the "New Math" as a parent, he is trying to see if his grandchildren can be spared the worst of the "New 'New Math.' "

Paul Nevai came to Ohio State from Budapest, via Leningrad, Orsay, and Madison. He works in orthogonal polynomials, difference equations, polynomial inequalities, and approximation theory.

His hobbies include bicycles, computers and computing, math history, and running. Since his son, Andrew, is a math major at Michigan, his favorite hobby is predicting the US math PhD job market in 2003.

[1]We refer the reader to the section "Answers to Some Frequently Asked Questions About the Hungarian Language" at the end of this article.

Adolf Szegő (father of Gabor Szegő).

had mentioned two papers of his which he said should be studied. I did not do it immediately, but three months later I did. One contained the solution of a problem I had been trying to solve for three years. His paper had been written 40 years earlier. I learned from this that when a great mathematician tells you to look at a paper which he or she thinks has been unjustly neglected, one should do it rapidly.

Gabor Szegő in 1896 (age around 18 months).

Hermina Neuman Szegő (mother of Gabor Szegő).

I (P. N.) only met Szegő once. It was in 1972 when I had just graduated. By that time, he had been inactive in mathematics research for almost a decade. Yet he was the mathematician who, for two reasons, had the greatest influence on my career as a research mathematician. One of the reasons was the book which we called *Pólya–Szegő*, that is, the problem book titled *Problems and Theorems in Analysis* by George Pólya and Szegő which needs no introduction for the readers of this article. The other reason was the book *Szegő*, that is, Szegő's monograph titled *Orthogonal Polynomials* and the accompanying contemporary theory of orthogonal polynomials, whose founding father was Szegő. To be really pedantic, he should be called the "founding grandfather," since it is already the third generation of mathematicians who is developing his theory today. These books will be discussed later in more detail. Very recently, I had an extraordinarily rewarding experience while working on a project which led to erecting Szegő's bust in Kunhegyes, St. Louis, and Stanford.

In telegraphic style: Gabor Szegő was Professor Emeritus at Stanford University. He was a member of the American Academy of Arts and Sciences, the Science Academy of Vienna, and the Hungarian Academy of Sciences. He was one of the prominent classical analysts of the twentieth century. He wrote more than 130 research articles and authored or co-authored 4 influential books, 2 of which were exceptionally successful. For analysts, Szegő is best known for *Szegő's extremal problem*, for his results on Toeplitz matrices which led to the concept of the *Szegő reproducing kernel* and which were the starting point for the *Szegő limit theorem* and the *strong Szegő limit theorem*, and for *Szegő's theory* of *Szegő's orthogonal polynomials* on the unit circle. These have been summarized in his books *Orthogonal Polynomials* (Colloquium Publications, Vol. 23, American Mathematical Society, Providence, RI, 1939) and *Toeplitz Forms and Their Applications* (jointly with Ulf Grenander, University of California Press, Berkeley and Los Angeles, 1958). The former is one of the most successful books ever published by the American Mathematical Society (four editions and numerous reprints). The book *Aufgaben und Lehrsätze aus der Analysis, vols. I and II* ("Problems and Theorems in Analysis"), which he co-authored with George Pólya in 1925, contributed to the education of many generations of mathematicians. It was first published by Springer-Verlag in the series Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen mit besonderer Berucksichtigung der Anwendungsgebiete (widely known as *The Grundlehren*) as volumes 19 and 20. The book *Isoperimetric Inequalities in Mathematical Physics* by Pólya and Szegő was published as No. 27 in the series Annals of Mathematical Studies by Princeton University Press, Princeton, NJ in 1951 (translated into Russian in 1962). Lawrence E. Payne writes on p. 39 of Vol. 1 of Szegő's *Collected Papers* (Birkhäuser, 1982): "Not only did this [book] make available to the mathematical public a number of powerful new tools of mathematical investigation but it also opened up an interesting new and fertile area of mathematical research." His work and results not only deeply influenced the development of pure and applied mathematics but also found many applications in statistics, physics, chemistry, and various fields of engineering science.

In what follows, we discuss Szegő's life and work, as seen by us and by several of his contemporaries.

After completing elementary school in Kunhegyes and graduating from high school in Szolnok (a town approximately 100 km southeast of Budapest) on June 28, 1912, he enrolled in the Pázmány Péter University in Budapest (today known as Eötvös Lóránd University), where he primarily studied mathematics and physics. The same year, he won first prize in the academic contest organized by the (Hungarian) Mathematical and Physical Society (which later became known as the Eötvös Competition and today is known as the Kürschák Competition). Winning the competition was much more than a passing event but rather a very important milestone in Szegő's career. The competition, as all mathematically inclined Hungarians know, carried a great deal of prestige. It was especially important for Szegő because it is doubtful that, as a Jew whose family had no connections, he would have been able without it to study or receive the attention that he did. His father had even tried to discourage him from entering the university since he thought that his son would have no future there as a Jew. Szegő later made sure his children knew of these circumstances. The following year, his paper on polynomial approximations of continuous functions received a University prize. He never abandoned approximation theory, and his very last research paper also focused on this subject.

Szegő spent the summers of 1913 and 1914 in Germany, first at the University of Berlin, later at the University of Göttingen. In Berlin, he attended the lectures of Georg Ferdinand Frobenius, Hermann Amandus Schwarz, and Konrad Knopp, and he also participated in Friedrich Schottky's seminar. In Göttingen, he took courses from David Hilbert, Edmund Landau, and a fellow Hungarian, Alfréd Haar, who was teaching there at the time.

When the First World War broke out, he immediately returned to Hungary and continued his university studies there until May 15, 1915. Conscription was underway and he knew that he would be drafted into the army of the Austro-Hungarian Monarchy. So, to avoid the infantry, he bought a horse and enlisted in the cavalry. According to his comments to his children, he was a poor horseman and fared poorly. He spent the last years of the war in Vienna. His military service lasted beyond the Austro-Hungarian capitulation on November 11, 1918; he remained in the army until early 1919. During this time, he served in the infantry, the artillery, and the air force. Naturally, the military applications of aeronautics were not very sophisticated at that time. However, the Austro-Hungarian Air Force had two extraordinary theoretical experts, Theodore von Kármán and Richard von Mises, two of the founders of modern aerodynamics. They both became Szegő's lifelong friends.

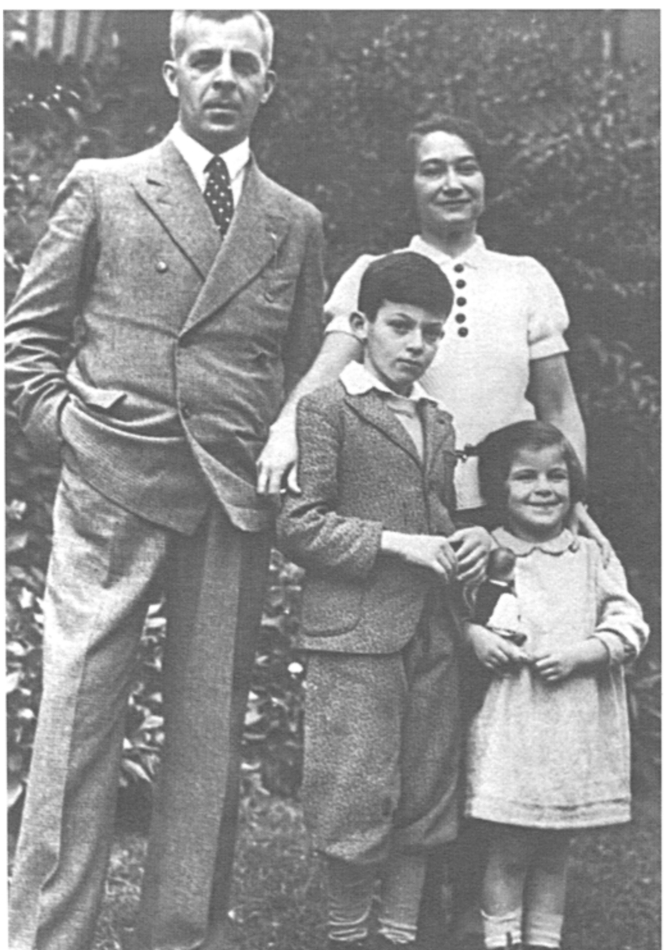
To avoid the infantry, he bought a horse and enlisted in the cavalry.

Between 1912 and 1915, Leopold Fejér, Manó Beke, József Kürschák, and Mihály Bauer were among his professors. He met George Pólya (who, at the age of 97, died in Palo Alto on September 7, 1985) and Mihály Fekete (of transfinite diameter fame) at this time; Szegő developed a long-lasting collaboration with both of them.

His first publication in an international journal, in which he gave a solution of a problem proposed by Pólya, was in *Archiv der Mathematik und Physik* 21 (1913), 291–292. As we know very well, there are problems at various levels. Some, like those done in school, everyone should learn how to do. Then there are contest problems, like those in the Mathematical Olympiads. These frequently require deeper insight than seems indicated at first reading. The problem of Pólya, which Szegő solved and published in 1913, is an example of a still harder type, which attracts prospective mathematicians. Hungary has long specialized in the use of problems to attract young students to mathematics; other countries have learned from them, and have contests of problems to encourage deeper mathematical thought.

His first research paper, "Ein Grenzwertsatz über die Toeplitzschen Determinanten einer reellen positiven Funktion," was published in the *Mathematische Annalen* 76 (1915), 490–503. This is how Pólya remembers it in 1982 (p. 11 of Vol. 1 of Szegő's *Collected Papers*):

> Our cooperation started from a conjecture which I found. It was about a determinant considered by Toeplitz and others, formed with the Fourier-coefficients of a function $f(x)$. I had no proof, but I published the conjecture and the young Szegő found the proof [. . .] We have seen here a good example of the fruitful cooperation between two mathematicians. Mathematical theorems often, perhaps in most cases, are found in two steps: first the guess is found; then minutes, or hours, or days, or weeks, or months, perhaps even several years later, the proof is found. Now the two steps can be done by different mathematicians, as we have seen.



Gabor Szegő, his wife Anna, and their children, Peter and Veronica, shortly after their arrival in the U.S. in 1934.

Szegő spent another 45 years working on sharpening, extending, and finding applications of the results published in this article, and the theory of Toeplitz determinants became one of his primary research areas. While he was serving in the military and his unit was stationed in Vienna, he received his Ph.D. from the University of Vienna on July 8, 1918. His dissertation was based on the above-mentioned article. Fifty years later, he returned to Vienna for a celebration of this, and I (R. A.) still remember how pleased he was recalling this celebration in conversation a few years later.

Szegő, having been a mathematical prodigy himself, was an ideal person to be asked to tutor one of the great mathematical minds of this century, John von Neumann (born as János Neumann in Budapest in 1903). Here is what Norman Macrae wrote in his book *John von Neumann* (Pantheon Books, New York, 1992, p. 70[2])

> Professor Joseph Kürschák soon wrote to a university tutor, Gabriel Szegő, saying that the Lutheran School had a young boy of quite extraordinary talent. Would Szegő, as was the Hungarian tradition with infant prodigies, give some university teaching to the lad?
>
> Szegő's own account of what happened was modest. He wrote that he went to the von Neumann house once or twice a week, had tea, discussed set theory, the theory of measurement, and some other subjects with Jancsi [Johnny in Hungarian], and set him some problems. Other accounts in Budapest were more dramatic. Mrs. Szegő recalled that her husband came home with tears in his eyes from his first encounter with the young prodigy. The brilliant solutions to the problems posed by Szegő, written by Johnny on the stationery of his father's bank, can still be seen in the von Neumann archives in Budapest.

Szegő was married on May 22, 1919, in Budapest just after he was released from the Austro-Hungarian Army. His wife, Erzsébet Anna Neményi, had a Ph.D. in chemistry from the Pázmány Péter University in Budapest. He was still in uniform when they were married. It is

[2]According to Macrae, the "coaching" took place in 1915–16, but most likely it was earlier. Macrae writes that von Neumann entered the Lutheran School in Budapest and that "László Rátz [was an] instructor in mathematics in Johnny's 1914–21." Then he writes that "Rátz's recognition of von Neumann's mathematical talents was instant [. . .] Rátz turned his student over to the mathematicians at Budapest University." This would suggest that the tutoring started in late 1914. On the other hand, Veronica Szego Tincher gathers from her father's comments that the tutoring occurred after the First World War", although it is also possible that their mathematical discussions began before the War."

said that during the ceremony, there was bombing from a boat on the Danube. They were to have two children: Peter (born in Berlin in 1925) and Veronica (born in Königsberg in 1929). Peter is an engineer by profession; he wrote a number of papers on special functions. He lives in San Jose where he is retired from work with the State Legislature of California. Veronica has lived in Southern California since 1954 and worked for the University of Southern California. She retired in 1995 as Executive Director for Budget and Planning and now lives in Palo Alto. Veronica has three children, Steven, Emily, and Russell. Emily has a son, Nathan, and Russell has a daughter, Micaela.

The Szegős lived in a happy marriage until Anna, after many years of suffering, died in 1968. Subsequently, Gabor married Irén Vajda in 1972 in Budapest. She died in 1982 in Budapest.

Turbulent revolutionary, counterrevolutionary, and anti-semitically discriminatory years followed the First World War (in political terms: Mihály Károlyi = middle, Béla Kun = left, and Miklós Horthy = right). There were only a very limited number of academic positions in Hungary. As a result, a great many Hungarian scientists who were not appreciated or were even labeled as unreliable characters in their own country, left Hungary, primarily for Germany, Switzerland, the United Kingdom, and a decade or so later, for the United States, where they received much more scientific and financial respect and reward than they could ever hope for in Hungary. For a short while, Szegő worked as an assistant of József Kürschák at the Technical University of Budapest in 1919 and 1920. After he could no longer work at the university, John von Neumann's father Maximilian helped Szegő.

Giving up all hope that he would ever get a job guaranteeing a reasonable living in Hungary, he moved to Berlin in 1921, where he became a friend and colleague of Issai Schur and worked with Leon Lichtenstein, von Mises, and Erhard Schmidt. For a result on the equiconvergence of orthogonal polynomial series and trigonometric Fourier series, he received his Habilitation in 1921. With this, he became a Privatdozent at the University of Berlin in May 1921. This meant that he had the right to give lectures but received very little compensation for it. Other mathematicians holding this title at the University of Berlin in the 1920s were Stefan Bergman, Salomon Bochner, Eberhard Hopf, Heinz Hopf, Charles Loewner,[3] and von Neumann.

From 1925 he had the *Lehrstuhl für angewandte Mathematik* (chair for applied mathematics). This was a *nichtbeamtete ausserordentliche Professur*, that is, an associate professorship without tenure. When Szegő left Berlin, Adolf Hammerstein (1888–1941) became his successor and held this position from 1927 to 1935. Szegő's above-mentioned paper was published in the *Mathematische Zeitschrift* 12 (1922), 61–94. At the same time, independently from Szegő, his Berlin colleagues Bergman and Bochner laid down the foundations of a theory of orthogonal functions that approached the problem from a different perspective. During this period, Szegő was also helping Lichtenstein with editing the *Jahrbuch über die Fortschritte der Mathematik.*

While in Berlin, he was awarded the Julius Kőnig prize by the Eötvös Lóránd Mathematical and Physical Society on April 10, 1924. The members of the prize committee were József Kürschák (president), Gyula Farkas, Dénes König, and Frederick Riesz. F. Riesz was asked to make a presentation report on the work of the recipient. His report was published in Hungarian in *Mathematikai és Physikai Lapok* 23 (1924), 1–6, and later it was reprinted (pp. 1461–1466) with a French translation (pp. 1573–1576) in the second volume of F. Riesz's *Oeuvres Complètes* (Akadémiai Kiadó, Budapest, 1960). We recommend it highly: an English translation is provided at the end of this article.

The two volume Pólya–Szegő is the best written and most useful problem book in the history of mathematics.

There is general consensus among mathematicians that the two-volume *Pólya–Szegő* is the best written and most useful problem book in the history of mathematics. In the 70 years since its first publication, it has continuously influenced mathematics research and made a great impact on the education and training of young mathematicians. So far, it has had four German, one English, one Hungarian, and three Russian editions. Both authors believed that mathematics could only be learned by *doing* mathematics. The book introduces the reader to mathematical research through a series of carefully selected and related problems, in such a way that after analyzing and solving a group of these problems, the reader is almost ready to do independent research in that particular area. Even though the title suggests that the book is about analysis and most of the problems were indeed selected from that area, a variety of problems from number theory, combinatorics, and geometry were also included, along with a few physical applications. The selection of the problems demonstrates the refined taste and mathematical elegance of the authors as well as their technical repertoire. Virtually every page offers something unexpected—an elegant argument, an unexpectedly clever proof, or a single problem that grows into a complex theory right

[3]He was called Karl Löwner in those days. In 1984 Louis de Branges proved the Bieberbach conjecture, the most famous previously unsolved problem in classical complex analysis, utilizing, in addition to the results of Milin–Lebedev and Askey–Gasper, the works of Loewner from those Berlin years.

in front of one's eyes. Pólya describes the origins of *Pólya–Szegő* (p. 11 of Vol. 1 of Szegő's *Collected Papers*);

> I cannot remember how and when the plan emerged; what is certain is that we worked on this plan for many years until the work appeared in two volumes in 1925.
>
> It was a wonderful time; we worked with enthusiasm and concentration. We had similar backgrounds. We were both influenced, like all other Hungarian mathematicians of that time, by Leopold Fejér. We were both readers of the same well directed Hungarian Mathematical Journal for high school students that stressed problem solving. We were interested in the same kind of questions, in the same topics; but one of us knew more about one topic and the other more about some other topic. It was a fine collaboration. The book [*Pólya–Szegő*], the result of our cooperation, is my best work and also the best work of Gabor Szegő.

It is hard to argue with Pólya's assessment of *Pólya–Szegő*. They set a standard for later books of problems, and no one has yet come close to their level. Szegő was always modest when he talked of this collaboration. He emphasized that the idea and the planning came from Pólya, and cooperative efforts followed.

Szegő was invited to the University of Königsberg to succeed Knopp in 1926 and worked there as Ordinarius (Professor) until 1934. His first two Ph.D. students were in Königsberg. One of his favorite stories is related to Hilbert's visit to the city in 1930. Hilbert was to be given an "hononary citizenship" by the city of Königsberg. He had not come dressed for the exceptionally cold fall weather. Szegő helped Hilbert out by lending him an overcoat so that his native city could welcome him.

Life became increasingly difficult for Jews in Germany in the 1930s. Szegő was one of the last to suffer, because he was so highly respected by his students and colleagues and because of his service in the First World War. This is what Pólya wrote to Jacob David Tamarkin from Zürich dated February 14, 1934:

> It was very difficult to write about the chief point which is the fate of Szegő. Well, I shall be brief and plain. I am terribly worried about him. I saw Mrs. Szegő in December. I got a letter from Szegő in the beginning of January; although no official measure was taken against him [until the beginning of January] and no direct collision happened with the students, I cannot see how it would go on indefinitely under those circumstances. He would accept, I understand, any offer even for a short period of 1 or 2 years, he should try to get a leave of absence for that time, and see whether he can live with his family on that amount. There is no hope to get something for him in Hungary, say Fejér and also Szegő himself [. . .] I could not do anything for him here in Switzerland [. . .] Excuse this letter, but you see, I am worried. The whole European situation is very dark.

Tamarkin set to work immediately, trying to find a job for Szegő in the United States (cf. p. 2 of Vol. 1 of Szegő's *Collected Papers*). Obviously, such a task was not simple in the middle thirties during the Great Depression; it was next to impossible even for American mathematicians to find jobs. Most positions involved a considerable amount of teaching (12 hours a week was not at all exceptional, and there were cases when it was even more) and were poorly paid ($3000 a year was considered a good salary). It is less widely known that Jews were not particularly welcome in the United States during the 1930s either. United States officials were seemingly not interested in providing asylum. Aided by the unrelenting support of some American mathematicians, quite a few Jewish mathematicians managed to come, and this played a major role in the development of mathematics in the United States. For a better understanding of the period, we suggest several articles published in *A Century of Mathematics in America, Part I* (Peter Duren, ed., American Mathematical Society, 1988); in particular, "The European mathematicians' migration to America" by Lipman Bers (pp. 231–243) and "Refugee mathematicians in the United States of America, 1933–1941: Reception and reaction" by Nathan Reingold (pp. 175–200).

In May 1934, during his Pentecostal holidays, Szegő went to Copenhagen to confer with Harald Bohr about his future. Without having to worry about German censors, he used this opportunity to write a letter, dated May 23, 1934, to Tamarkin. The background was this. In 1925, Szegő had been invited for a visiting appointment to Dartmouth College. The Szegős considered it carefully but decided to turn it down because his future seemed more secure in Germany. Szegő then recommended Tamarkin for the position. It is thus not surprising that Szegő turned to Tamarkin for assistance; they continued to be close friends until Tamarkin's death in 1945. Szegő's letter to Tamarkin was written in German; it is reprinted on pp. 3–6 of Vol. 1 of Szegő's *Collected Papers*.

In this letter, he describes his feelings about the future of his family and his contemporaries. He seems unaware of how serious the situation was. In reality, it was their lives that were at risk. (According to Carl de Boor, who translated the letter for us, Szegő's German style was excellent and it would be hard to recapture it in English.)

> Copenhagen, 23. May 1934
>
> Dear Mr. Tamarkin!
>
> For some time now, I have been planning to write to you and thank you, respectively Professor Richardson, most cordially for all the efforts you have made on my behalf. Please forgive the fact that, once again, I write in German, I can in this way express myself partly more easily, partly more precisely. I do hope that the reading of this letter will not be difficult for you because of the language.
>
> I have come to Copenhagen for a few days over the Pentecost break. I will tell you in a moment what made me make this journey. However, in the interest of clarity, I want to begin with a short description of my situation, starting roughly at the point last summer when we last corresponded concerning these questions. Since that time, there has been, on the face of it, no essential change in my personal situation. I have been treated, by colleagues as well

as students, correctly and, despite the present political passions, bearably. Nevertheless, my situation continues naturally to be very difficult, in many instances very depressing and offensive. At the center of these difficulties stands the worry about my family, especially the education and future of my children. In order keep him away from the politics-filled air of the school and the life in Germany, we have, already last fall, sent our nine-year old boy to Switzerland, where he is well taken care of, body and soul. Since a few days ago, he is with us during summer vacation, giving us the opportunity to see clearly the great advantage of his stay there. However, this situation cannot be maintained for long.... The future of our children is hard to visualize in Germany. This, I am convinced, would be the case even if, against expectations, there were to be a change in the general direction taken by the government....

Added to these considerations concerning the present and future of my children is the point that I have no faith at all in the stability of my own situation. Last summer, we had many discussions with Fejér about these matters, considered parallels to the analogous (as we saw it) development in Hungary, etc. Some of the prophesies have already been contradicted by what has happened. In Germany, the course of the new 'Weltanschauung' is being maintained with such single-mindedness that a change, a compromise, or even an attenuation in the near future cannot be expected. Of course, there are shades and differences of temperament in the leading circle, and it is impossible to predict which forces will finally be victorious. For example, in the recent formation of the "Reichskultusministerium' [Ministry for Culture] (a change which is of prime importance for personnel questions at the universities), the better spirit seems to have come out on top. Yet, one still hears reassurances that in 5 years no 'non-Aryan' person will occupy a university chair. I point to the many retirements that have taken place recently, often without any proper procedure (e.g., Rademacher, at the end of February of this year), also to retirements for the sake of administrative simplification, but with the hidden goal to remove unwanted persons who otherwise would be protected by the 'Beamtengesetz' ['Law for the Restoration of the German Civil Service']. Just a few weeks ago, an outstanding classical philologist has in this way been removed from my university. In Mathematics, the situation in Königsberg is as follows. Reidemeister has been moved, and no successor has so far been appointed. This means that I am needed *for the time being* since I am alone in a position of responsibility. However, as soon as the successor arrives, something to be expected rather sooner than later (probably by the fall), I don't expect to stay around much longer, even though, as a participant in the war and officer at the front, I supposedly am not affected by that bill. Last summer, it was generally thought that this 'Beamtengesetz' would be temporary in any case, so sooner or later the [former] law-based security would be restored. Since then, the bill has been extended twice already, and there is nothing to prevent further extensions *ad inf.* In addition, should the bill be revoked, there remain a thousand other means for making it impossible to work here. Such bills are changed with much greater ease than, say, a mathematician would switch from one system of axioms to another.

One other pertinent fact deserves to be stressed. Königsberg has been called, semiofficially, a 'Reichsuniversität', meaning that in future only politically correct people will work there. It is therefore nearly impossible that I will remain there after the final arrangements have been made, probably this coming fall, as a consequence of the formation of the Reichskultusministerium. Rather, if not pensioned off at once, I will be moved to a different university. This used to be impossible for university professors in Germany; today, the 'law' provides the means for it! Now, what can I expect of colleagues and students in a new environment, likely to be of ill will toward me from the start? Probably, they will only see a person who has been moved as punishment and will hardly tend to allow for mitigating circumstances, of the kind I am used to here in Königsberg, where one knows me from former times when judgements were still objective and, with the slogan 'There are decent Jews, a rare exception' as a kind of excuse, behaves correctly toward me.

All in all, my situation is, from the standpoint of my children, equally bad for present and future, but my own future is extremely uncertain. The longer one waits, the more difficult is the change to a new milieu likely to be. For this reason, I continued to think, after our correspondence last year, of moving to the U.S.A. In spite of my rather hopeless situation, just described, I do not wish to proceed with this hastily; in particular, I am not forgetting at any moment the difficulties which exist over there and which you kindly described last year to me and my friends Fejér and Pólya. Therefore, we arrived with Fejér and Pólya at the conclusion that I will continue to look for a position in the U.S.A., but that in case of success.... I shall try first to get temporary leave from here in order not to burn any bridges.

It was in this sense that I asked Pólya last fall to write to you. On 7. April of this year, he told me that you had responded and had related the results of ca. 20 written inquiries. I am really moved by this undeserved measure of willingness to help! In addition, he asked me to let you know exactly, through him, my own intentions, since you would have to act quickly and decisively. For the sake of clarity, I am stating these today again, even though you are certain to have had his answer for some time now. Here is my thinking: In case A), if

1) the offer in question for 2–3 years is certain, and there is some hope of an extension,

2) if it is such that I and my family can live on it, then I would accept it *for sure*. However, in case B), if the two conditions just formulated are not fully satisfied, I would apply for temporary leave, and make further decisions only after I have that leave. Of course, I would also apply for such leave in the case A), but that wouldn't be so utterly important as in the case B).

In a letter from 9. May, Fejér tells me, based on a letter from you, that Washington University in St. Louis plans to offer me a visiting professorship for 3–4 years, assuming that it can obtain the necessary financing, which probably is not certain. He also writes that you, in order to proceed, would like to know whether, in case of an official invitation, I would be able to obtain leave from my university.... I decided on a moment's notice to travel to Copenhagen in order to speak with Bohr in person. I am now there, staying with him, and we are discussing the situation day and night. He kindly showed me the letter from Richardson; by the way, in a telegram two days ago he has announced my presence here as well as a letter to be sent. These happenings explain my somewhat late but, so I hope, very much clearer response than would have been possible from Germany. In any case, I ask for your and Prof. Richardson's indulgence, should this lateness cause the postponement of some steps....

I now would like to ask you, dear Mr. Tamarkin, to understand this response, as well as inform the relevant people, as if the possibility of an official leave were already settled. As a matter of fact, I do consider this very likely. However, I am going one step further. Since in the matter of St. Louis, the above-mentioned condition A), 1) seems to be satisfied, I would accept the offer irrespective of the leave, provided also condition A), 2) is satisfied. In this respect, I am however badly informed, in particular, I have no idea what is the sum needed in the U.S.A. to keep a professor with family from starving to death. I have no intention of making special demands. In order to have a basis for comparison, I mention that my present yearly income is about 9000 M., but it is likely to experience increasing diminution because of the present uncertain situation. However, I hardly believe that it makes sense to convert this into U.S. currency, especially since its devaluation. In any case, I would be very grateful if you could give me some information on this point. About St. Louis itself, I couldn't find out much: It is about 1,400 km from New York, an industrial city with 800,000 inhabitants, several universities, with the one in question apparently well equipped and financed (cf. *American Universities and Colleges*, 2nd ed., Amer. Council on Educ., Baltimore, 1932), however the mathematicians there are unknown to me. This wouldn't bother me at all. It would, however, be very valuable to me if I could learn from you also about city and university, also about climate, housing possibilities, and standard of living. I am assuming that you are able to say something based on hearsay or even on personal experience.

I have no idea what is the sum needed in the U.S.A. to keep a professor with family from starving to death.

I dare to stress one *very important* point. A leave of absence longer than 1 year for me cannot be hoped for at present. (I can apply for an extension later.) On the other hand, I can't really apply for a 1-year leave of absence based on an invitation for 3–4 years, as was the formulation transmitted to me by Fejér, extremely valuable though it is to me. Should the invitation take this form, then I would have the additional big and important request to you, to inform the relevant office about my situation and to request that, in addition to an invitation for 3–4 years for my personal use, a second invitation be sent in which *only* the duration of 1 year is mentioned. I hope that I have managed to be completely clear on this particular point.

Don't be surprised when you receive very soon a paper of mine in English. I corresponded with Professor Walsh about a certain question and afterwards wrote up my results in English. He then asked me to send him the paper and he is going to honor my request to send it on to you or Professor Hille. I hope that the editing won't be as bad as when Japanese write in German. By the way, I am studiously learning English and hope to master quickly the art of giving talks (if not the art of day-to-day conversation), once I have had for a while the opportunity to listen to, resp. give, classes in English. In order not to disappoint the gentlemen in St. Louis too much, it might be advisable to mention on occasion my low skills in this area.

I now close, having, I hope, developed the essentials clearly. With repeated warmest thanks to you and Professor Richardson, I remain, with cordial greetings,

sincerely yours,

[signed] G. Szegő

At the same time, H. Bohr also wrote a letter to Tamarkin, explaining that Szegő's situation in Germany "is still more untenable than he pictures it himself." He also writes that the fact that "Szegő until now has been able to maintain his position, is only due to his quite exceptional position among his pupils, because he not only is a first class mathematician, but also an extremely estimated, inspiring and successful teacher." H. Bohr's letter is reprinted on pp. 2–3 of Vol. 1 of Szegő's *Collected Papers*.

Tamarkin's efforts were successful and Szegő was offered a professorship at Washington University in St. Louis, Missouri, in 1934. These were extraordinary times both economically and politically. The university did not have sufficient funds for Szegő's salary. The money was raised by a grant of $4000 from the Rockefeller Foundation, by a matching grant from the Emergency Committee in Aid of Displaced German Scholars, and from donations from the local Jewish business community, which covered Szegő's salary for four years. The Rockefeller Foundation was also instrumentally involved in arranging visas, exit permits, travel documents, and so forth.[4]

Following the advice of Pólya and H. Bohr, Szegő accepted the job, went to St. Louis in the fall of 1934, and remained there until June, 1938. In the course of that time, he was the advisor to five Ph.D. students (one of them only graduated in 1948 at Stanford) and finished the first version of his book *Orthogonal Polynomials*. He had a summer visiting appointment at Stanford University in 1935. In 1936, he gave an invited address at a meeting of the American Mathematical Society. The friendships and contacts he made in St. Louis lasted to the end of his life.

Orthogonal Polynomials was first published in 1939, and it has since become one of the main reference books for many pure and applied mathematicians and for scientists working in various fields. There are many reasons why researchers started investigating orthogonal polynomials. Historically, these polynomials first appeared in connection with special functions, numerical analysis, and approximation theory (quadrature and interpolation). They are also denominators and numerators of convergents of continued fractions. Later, they would arise in Padé approximations and moment problems.

However, the foundations of a general asymptotic theory of orthogonal polynomials were first laid down in a series of papers that Szegő wrote in the 1920s and 1930s. Szegő succeeded in reducing many significant problems related to orthogonal polynomials to the asymptotic behavior of certain Toeplitz and Hankel determinants. This ingenious result made the solution of many problems easy, at least in the case of the so-called *Szegő class*, that is, for measures whose absolutely continuous component is Lebesgue integrable on the unit

[4]P. N. appreciates very well the utmost significance of the latter.

circle. Many mathematicians contributed to developing *Szegő's theory*, including Naum I. Akhiezer, Sergey N. Bernstein, Paul Erdős, Géza Freud, Yakov L. Geronimus, Alexander N. Kolmogorov, Mark G. Kreĭn, James Alexander Shohat, Vladimir I. Smirnov, and Paul Turán, just to name a few. Szegő's theory has found significant applications in other scientific fields such as numerical methods, direct and inverse discrete scattering theory, differential and difference equations, mathematical statistics, prediction theory, statistical physics, systems theory, coding theory, and fractals. A theory of orthogonal polynomials beyond Szegő's class was only recently discovered. The primary reason for the renewed interest lies in the various recurrence formulas satisfied by orthogonal polynomials. Szegő's *Orthogonal Polynomials* discusses almost all facets of the theory, including those areas that (often inspired by Szegő's book) were to be developed only later. Quite a few well-written books have appeared on orthogonal polynomials since; yet Szegő's remains the standard, the first place to look for ideas and information.

In 1938 Szegő accepted an offer from Stanford University to become Head of the Department of Mathematics there. He was the Head until 1953. His greatest achievement during this time was to raise Stanford's mathematics to a world-class standard. At this point we turn the pen over to Peter Lax by quoting from "The old days" to be published in *A Century of Mathematical Meetings* (Bettye Anne Case, ed., American Mathematical Society, 1996).

In the forties and fifties I spent many summers at Stanford University at the invitation of the Head of the Mathematics Department, Gábor Szegő, who was my uncle by marriage. The Head of a department was in those days a much more powerful figure than a mere chairperson today; he made all decisions, including hiring and firing. Szegő used his powers to turn the provincial mathematics department that Stanford had been under [Hans Frederick] Blichfeldt and [James Victor] Uspensky—both remarkable mathematicians—into one of the leading departments of the country that Stanford is today. He appointed four senior mathematicians from Europe: Pólya, Loewner, [Max M.] Schiffer [who succeeded Szegő as head of the Department] and Bergman, and half a dozen brilliant young Americans: [Richard] Bellman, [Albert Hosmer] Bowker, [Paul] Garabedian, [Halsey] Royden, [Albert Charles] Schaeffer and [Donald C.] Spencer. He took advantage of the availability of postwar Government support for science by joining Bowker in the creation of the Applied Mathematics and Statistics Laboratory.

The Szegő period at Stanford is well documented in Royden's article "A history of mathematics at Stanford" in Part II of *A Century of Mathematics in America*, published by the AMS, [1989, pp. 237–277]:

He wrote his first paper [. . .] at age 20 while in the trenches during the First World War. His comrade-in-arms and later lifelong friend, Strasser, recalled that his fellow officers realized that Szegő was a very precious talent, and did their best not to expose him to danger.

As a young man, Szegő was very shy; by the time he came to the United States, he was a self-assured man with old world courtly manners. Underneath his somewhat aristocratic appearance he was a warmhearted person, ever willing to help others. He was aware of the absurdities of life and savored them. He had high standards, but did not expect everyone to live up to them.

Throughout his scientific life, he was very close to his mentor, George Pólya. Contrary to a mistaken assertion in a recent biography of Pólya by the Taylors, Szegő always treated him with utmost consideration and tact. The two had different personalities; Pólya was conservative and pessimistic, Szegő liberal and optimistic. To illustrate their differences with a small story, when Marcel Riesz came to Stanford, Pólya felt he couldn't invite to his house a man who had sired two illegitimate daughters; Szegő was amused by this Victorian prudery.

Szegő believed that one should do mathematics as long as one can; he made Pólya agree to delay his study of the psychology of problem solving until the age 65. Since this left Pólya almost 30 years for his educational enterprise, it was not a bad bargain.

[. . .] in 1946 there was one departmental secretary at Stanford. All faculty members, including Szegő, typed their own papers. All this changed soon in the postwar boom, but there was a corresponding loss of intimacy. For instance, it would be impossible today, as Szegő did in the summer of 1946, to invite all graduate students to his home for a supper of stuffed cabbage and plum dumplings, cooked expertly by his wife.

Cooking wasn't Mrs. Szegő's only expertise; she was a chemist, and supported the family while Szegő served in the prestigious but barely remunerated position of Privatdozent in Berlin. She was a voracious reader, in four languages, and provided intellectual companionship and stimulation to her husband, and their children. It was a happy household, and I was privileged to be part of it for three summers.

Szegő stayed at Stanford until his retirement in 1960 as Professor Emeritus.

Peter Duren recalls:

When I was an Instructor at Stanford, I attended Szegő's course on orthogonal polynomials. It was really a course on various techniques in analysis: asymptotic estimation of integrals, for instance. I remember that he needed to use a Blaschke product at one point, but he didn't call it that. When one of the students asked whether that was a Blaschke product, he replied that some people called it that, but he would not want to honor that man in any way. The student immediately caught on and asked whether Blaschke had been a Nationalist Socialist. Szegő didn't respond directly, but the gleam in his eye confirmed that the student had got it right.

Times have changed. Today, it's no big deal to call Blaschke a Nazi and still talk about Blaschke products. We do it all the time. We wonder how Szegő would have reacted to the solution of the Bierberbach conjecture.

Bob Osserman recalls:

I can recall one Szegőism from a conversation I had with Szegő at a party shortly after I arrived at Stanford. He said to me something to the effect: *Don't you think it's somewhat fraudulent that we claim to teach people how to become research* *mathematicians? That's like claiming you can teach someone how to become a poet. All you can really do is show by example how research in mathematics is done, and then they either can do it themselves or they can't.* At the time, I thought that was a fairly shocking idea, perhaps meant deliberately to be a bit outrageous, but I gradually came to realize that there was more truth to it than I would have at first conceded.

Don't you think it's somewhat fraudulent that we claim to teach people how to become research mathematicians? That's like claiming you can teach someone how to become a poet.

In his article already cited, Halsey Royden tells about Marcel Riesz's series of four lectures at Stanford in 1948:

> The day of the first lecture was warm, the good-sized lecture room was full of faculty and students. Gabor Szegő introduced Riesz, who promptly took off his jacket and proceeded to lecture in his shirtsleeves and suspenders. A bowl of water and sponge had been provided. After filling up the blackboard, Riesz motioned imperiously to Szegő, who jumped up and washed off the blackboard while Riesz stood by and watched! Now Szegő was very distinguished and autocratic; wore elegant tailor-made suits, and was always regarded with awe by the students and most of the faculty. To see him in the role of young European assistant to Riesz was startling! After several repetitions of this performance, needless to say, blackboard and floor soon became quite a mess. Sitting directly behind me was George Pólya, who had brought Felix Bloch to hear a distinguished fellow Hungarian. Pólya was somewhat embarrassed by the performance and muttered apologies *sotto voce*.

Szegő became a naturalized American citizen in 1940. In 1945–1946, he spent a year teaching mathematics to American soldiers (waiting to be shipped back to the States) at the American University in Biarritz, France. He served as a civilian employee of the War Department; he was in uniform and was given a rank equivalent to Colonel with PX and similar privileges. Szegő's son, Peter, was serving in the U.S. Army at the same time and was also a student in Biarritz while Szegő was teaching there. During this time, Szegő traveled to the Netherlands and England where he met and assisted various mathematicians. Joseph Ullman (Professor Emeritus at the University of Michigan, who died on September 11, 1995, at the age of 72, while we were putting the finishing touches on this manuscript) met Szegő in Biarritz and later followed him to Stanford, where, under Szegö's direction, he defended his Ph.D. in 1950. Michael Aissen (Professor Emeritus at Rutgers University, Newark) and Robert L. Wilson (Professor Emeritus at Ohio Wesleyan University) were also students of Szegö in Biarritz. Szegő also tried to get permission to enter Hungary, but the Russians declined his request. His mother lived in Budapest until she died there in 1946. At the time Szegő was in France, the Szegős knew little of what happened to his brother and his wife's family.

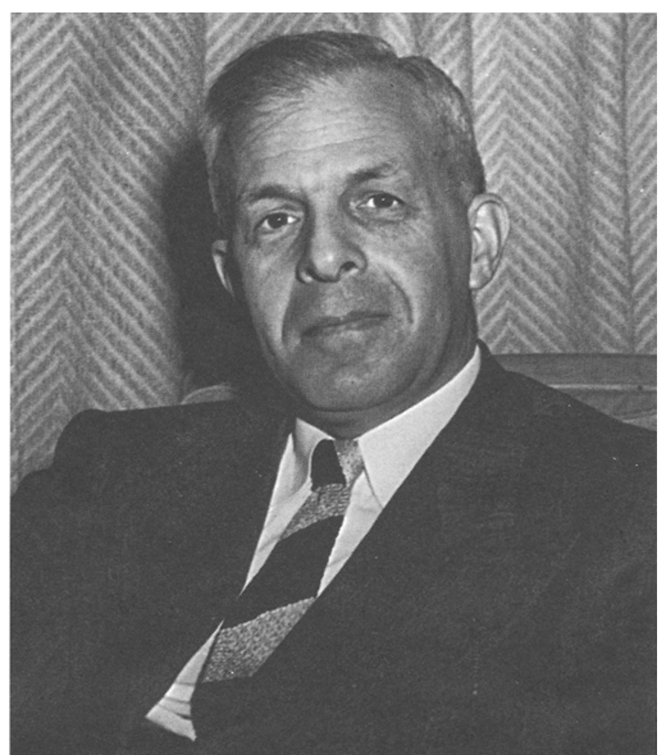
Michael Aissen recalls:

> The students [in Biarritz] became aware of Szegő's prestige because of a curious fact. The faculty had a veneer of democracy evidenced by the fact that the standard form of address was Mister. . . . However, there was a single exception. Everyone addressed Gabor as *Dr. Szegő*.

Szegő received a lump sum reparation from the German government during the 1950s. When he reached retirement age, he received the equivalent of the pension he would have had if he had not been forced to leave Germany.

After he retired, his wife's health deteriorated and Szegő's health declined as well. He gave his last mathematical talk, on Fejér's work, at an international conference on Constructive Function Theory in Budapest in 1969. Anna died in 1968 and, in 1970, Szegő discovered that he was suffering from Parkinson's disease. During the years between 1973 and 1980 he split his time between Palo Alto and Budapest (often at the Grand Hotel on Margaret Island on the Danube). When he was in Budapest, his friends and followers often visited him. He particularly enjoyed the company of György Alexits, Erdös, László Fejes Tóth, and Turán. During his last years he was confined to a wheelchair and suffered a lot of pain.

Even after Szegő stopped doing mathematics research, papers, problems, and questions continued to be sent to him from all over the world. He was pleased to



Gabor Szegő, around 1950.

receive them, but it annoyed him that he could not respond to all of them as he once had.

In 1952, Szegő published an extension of his first paper titled "On certain Hermitian forms associated with the Fourier series of a positive function" (*Comm. Sém. Univ. Lund*, Tome Suppl., Festskrift Marcel Riesz, 1952, 228–238). About this paper Barry McCoy wrote on pp. 47–52 of Vol. 1 of Szegő's *Collected Papers*:

> It is easily arguable that, of all Szegő's papers [this] has had the most applications outside of mathematics. In the first place, the problem which inspired the theorem was propounded by a chemist working on magnetism. Extensions of this work made by physicists have led to surprising connections with integrable systems of nonlinear partial difference and differential equations [. . .] In addition Szegő's theorem has recently been used by physicists investigating quantum field theory and Toeplitz determinants arise in the study of static monopole solutions of Yang–Mills equations.

One way mathematicians are honored is to have something they discovered named after them. As we mentioned before, he made a number of such discoveries. Another way we show that the work of mathematicians is deep enough to last is to publish their selected or collected works. He was still alive when his collected works were published in three thick volumes (2626 pages) in 1982 with the title *Gabor Szegő: Collected Papers* by Birkhäuser in its series Contemporary Mathematicians. I (R. A.) sent a copy of the three volumes to Szegő via his son Peter who looked at it before taking it up to his father. Szegő had a very good day when Peter brought it; he was not only very pleased, but he kept also asking Peter if he had seen this in the book, and then going on to ask about something else he saw there.

Mark Kac wrote in his review of Szegő's *Collected Papers* published in *The American Mathematical Monthly* 91 (1984), 591–592:

> For who could be indifferent to the theorem that a power series with only finitely many different coefficients either represents a rational function or is not continuable beyond its circle of convergence! Or if a Toeplitz matrix is generated by the Fourier coefficients of a nonnegative Lebesgue integrable 2π-periodic function f, then the nth roots of the determinants of the $n \times n$ truncated matrices converge to the geometric mean of f. [The last sentence is a paraphrase of what Kac wrote with formulas.]
>
> I am picking these two examples, both from Szegő's early years, because the first one is one of the many isolated jewels scattered throughout the books, and the second the beginning of an important development which is likely to continue for many years to come.
>
> It is characteristic of most, if not all, of Szegő's work that it begins with a concrete problem. That much of it flowered into elegant general theories (e.g., orthogonal polynomials on the unit circle) is a tribute to Szegő's impeccable taste in choosing problems and to the depth of his insight. Even then no one, not even Szegő himself, could have dreamed of the extent to which some of his work would ultimately influence mathematics and science [. . .]
>
> They are a monument to the vitality of classical analysis and to the virtuosity of their author.

Szegő left a memorial for us, his mathematical work. It continues to live and lead to new work. We often regret that he is not here to appreciate all of the work being done on problems he started.

Epilogue

The city of Kunhegyes celebrated the Szegő centenary on January 21, 1995. An entertaining description of the day's event, including two speeches by Lee Lorch and R. A., can be found at URL http://www.math.ohio-state.edu/JAT/DATA/SPECIALS/szego.

With the help of generous contributions by over 100 individuals, the city of Kunhegyes, Washington University, and Stanford University, Szegő's bronze bust was commissioned from the Hungarian artist Lajos Györfi. The dedication of the statue, which was erected in front of the City Library in Kunhegyes, took place on August 23, 1995. A short account of the dedication ceremony by Kathy A. Driver can be found at URL http://www.math.ohio-state.edu/JAT/DATA/SPECIALS/szego.bust. In addition, copies of the bust will be placed at Washington University and Stanford University.

Answers to Some Frequently Asked Questions About the Hungarian Language

First, a short course in pronouncing Hungarian words. It is the easiest thing on the face of the earth.[5] Hungarian is almost exclusively phonetic; that is, words are pronounced exactly as they are written (with a few exceptions). The (single) letter "sz" stands for "s" as in "set." The letter "s" is pronounced as "sh" as in "sheet." The letter "ö" is just like its German twin: "Hölder." On the other hand, "ő" is a looong "ö" as in "Szegööö." Incidentally, "z" is as in "zero," and the (single) letter "zs" is pronounced as in the French "Legendre." Surprise: the letter "á" is not a long version of "a"; it's a vowel in its own right, as in "art." Homework: say *Zsazsa Gábor*. N.B. *Gábor* is legitimate both as a family and a given name.

Note that Szegő is also frequently spelled as Szegö or Szego. In addition, Frigyes = Frederick (as in Riesz), Gábor = Gabor (as in Szegő), György = George (as in Pólya), Gyula = Julius (as in Kőnig), Lipót = Leopold (as in Fejér), Marczel = Marcel (as in Riesz), Miksa = Maximilian (as in John von Neumann's father who was also called Max), Pál = Paul (as in Erdős and Turán), and Tódor = Theodore (as in von Kármán).

The custom in Hungarian is that family name comes first and given names follow.

[5]Come now. Pronouncing Czech must be just as easy.—*Editor's Note.*

Acknowledgments

We thank Veronica Szego Tincher and Peter Szego for information of a personal nature and for their continued support of this project. We thank Carl de Boor for consulting on German-related matters in this article and for his translation of Szegő's letter to Tamarkin. We thank Dietrich Braess and Herbert Stahl for helping us to figure out how the German academic system worked in the 1920s. We thank Priscilla R. Feigen for helping us locate some former members of the Department of Mathematics at Stanford University. We thank Edit Kurali for translating the article "Gábor Szegő" by P. N. which was published in Hungarian in *Magyar Tudomány* 8–9 (1986), 728–736 [and was reprinted as "Hungarian scientists, XVI: Gábor Szegő" in *Nyelvünk és Kulturánk* 65 (1986), 57–63]. We thank Peter Lax for allowing us to use an excerpt from his not yet published article. We thank Michael Aissen, Peter Duren, and Bob Osserman for anecdotes about Szegő. We thank László Szabó for translating F. Riesz's presentation report from *Mathematikai és Physikai Lapok* 23 (1924), 1–6. We thank Liz Askey, Chandler Davis, Peter Duren, Samuel Karlin, Peter Lax, Lee Lorch, Peter Szego, Veronica Szego Tincher, and István Vincze for reading a draft version of this work and suggesting many improvements. Parts of the present paper are based on the above article by P. N. in *Magyar Tudomány* which itself has greatly benefited from the introductory material to Szegő's *Collected Papers* edited by R. A. and published by Birkhäuser, Boston, in 1982.

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Appendix: Frederick Riesz's Report on the 1924 Gyula Kőnig Prize[6]

Gentlemen!

The committee entrusted with the task of making a proposal concerning the award of the second Gyula Kőnig Prize held a meeting on January 26 of this year at the Technical University with József Kőrschák as chairman; also present were Gyula Farkas, Dénes Kőnig, and yours truly.

The committee observed with pleasure that among the Hungarian mathematicians eligible, according to the rules of the foundation, there are more than one who are worthy of receiving the prize. This year the committee wanted to reward a member of the youngest generation and decided to recommend for the prize Gábor Szegő, Privatdozent at the University of Berlin.

The committee has charged me with the task of preparing a report and of analyzing and appraising the works of the candidate. I have the honor to present my report.

During his eight years of scientific activity Gábor Szegő has produced numerous works. Please permit me to restrict myself to those of his papers that attracted most of my attention by the novelty, beauty, and significance of their results and methods. From among those results, if you excuse me for my perhaps excessive subjectivity, I start with a discovery that is in direct contact with my own research. It is known and easy to prove that the value of a function that is holomorphic inside a curve, say, a circle, and continuous on an arc of this circle, cannot be constant on this arc except in the trivial case when the function is constant. In 1906 the French mathematician Fatou, after showing in his famous doctoral dissertation that every function that is bounded and holomorphic inside a circle has a limiting value almost everywhere, that is, with the exception of a set of measure 0, raised the following question: since this limiting function cannot be constant on the whole arc, as was stated above, how large can the set be on which it is constant; or, and this amounts to the same, how large can the set be on which it vanishes? After showing that this set cannot fill out "almost" entirely an arc, he formulated the conjecture, which he believed was difficult to prove, that this set has measure 0. My younger brother Marcel and I proved this conjecture in a joint article, which we presented at the 1916 Stockholm Congress, not only in the bounded case but for a more general class of holomorphic functions as well.

Szegő succeeded in showing the deeper, I could say real, reason of this phenomenon in a March 1920 letter addressed to me which was published, together with my comments, in the 38th volume of *Math. és Term. Értesítő* in 1920 under the title "Analytikus függvény kerületi értékeiről." Szegő later also published his related research in the 84th volume of *Math. Annalen* under the title "Über die Randwerte einer analytischen Funktion." Namely, in these papers he proved that the logarithm of the absolute value of the [nontangential] boundary limit function is Lebesgue integrable. Therefore, the logarithm may be equal to negative infinity, that is, the boundary limit function itself may vanish, only on a set of measure 0. The interesting nature of this result is perhaps better shown by the following theorem which is easily seen to be equivalent to it: given a nonnegative function on the circumference of a disk, a necessary and sufficient condition for the existence of a function that is holomorphic inside the disk and is not identically vanishing inside the disk and has bounded mean value, such that the absolute value of its [nontangential] boundary value is almost everywhere equal to the given function, is that both the given function itself and its logarithm be integrable.

I note that Szegő's theorem, which he obtained in a roundabout way via the study of Toeplitz forms and the Fourier series of positive functions, can very easily be derived from a famous formula of Jensen. This was pointed out not only by me in the above cited correspondence, but also by Fatou himself, who applied a similar chain of ideas to prove in just a few lines, not the theorem of Szegő, but his own conjecture.

Another, extensive group of Szegő's work belongs to the following sphere of ideas: from properties of the coefficients of power series, or from arithmetic properties of most of these coefficients, he deduces properties of the corresponding analytic functions. More specifically: (i) theorems of

[6]Translated from Hungarian by László Szabó (lszabo@sol.cc.u-szeged.hu) from *Mathematikai és Physikai Lapok* 23 (1924), 1–6 [cf. in Hungarian (pp. 1461–1466) and in French (pp. 1573–1576) in the second volume of Riesz's *Oeuvres Complètes*, Akadémiai Kiadó, Budapest, 1960].

Hadamard and Fabry about lacunary power series; (ii) the theorem conjectured by Pólya and proved by Carlson about power series with integer coefficients that are convergent inside the unit disk, which states that the function defined by such a power series either is rational or else cannot be extended beyond the unit disk; and (iii) the analogous theorem of Szegő about power series having only finitely many different coefficients ("Über Potenzreihen mit endlich vielen verschiedenen Koeffizienten," *Sitzungsber. d. preuss. Akademie*, 1922). In "Tschebyscheff'sche Polynome und nicht fortsetzbare Potenzreihen," which appeared in the 87th volume of *Math. Annalen*, Szegő shows that all these theorems follow naturally from the relationship discovered by Faber that exists between the Tschebyscheff polynomials of a curve and conformal mapping, and which was used by Carlson in his proof of Pólya's conjecture. In the same article, starting from the same principle, Szegő also deduces a theorem of Ostrowski which leads us to a seemingly distant theorem of Jentzsch, a young German mathematician who died in the war, about the distribution of zeros of the partial sums of power series. His article titled "Über die Tschebyscheff'schen Polynome," which was published in the first volume of mathematical *Acta* [*Acta Sci. Szeged*] of the Ferenc József University, and another one, a short article titled "Über die Nulstellen von Polynomen, die in einem Kreise gleichmässig konvergieren," which was recently published in the *Sitzungsberichte* of the Mathematical Association of Berlin belong to this area as well. In them Szegő, using completely elementary methods, throws light on the deeper causes behind the theorem of Jentzsch and related phenomena.

Finally, I turn to the area belonging both to complex and real analysis to which Szegő devoted the largest part of his work: the theory of orthogonal systems and the corresponding series expansions. To start with a smaller, very interesting work, which also shows his great ability in formal calculations, let me mention the article "Über die Lebesgue'schen Konstanten bei den Fourier'schen Reihen" which appeared in the 9th volume of the *Math. Zeitschrift*. Here he gives very simple numerical expressions for Lebesgue constants whose properties were previously studied by Fejér and Gronwall. Then he proves in a straightforward fashion properties of these constants some of which were proved by the above-mentioned authors in a much more complicated way and some of which were conjectured by them. In more extensive work which appeared in the same volume under the title "Über orthogonale Polynome, die zu einer gegebenen Kurve der komplexen Ebene gehören," he examines Fourier expansions in polynomials which are orthogonal on a closed curve. This contains, as a special case, both Legendre and power series. These expansions, even in the general case, behave very much like power series, and provide a new and most natural solution to the following problem of Faber: given a domain, find a system of polynomials in which every function that is holomorphic in this domain has an expansion. The expansions studied by Szegő have an interesting and very simple relationship with the conformal mapping of the finite and infinite domains bounded by the given curve onto the unit disk. For example, the conformal mapping between the exterior of the curve and the exterior of the unit disk which maps infinity onto itself is the limit of the ratios $P_{n+1}(z)/P_n(z)$ formed from consecutive polynomials.

Among the papers of Szegő related to the problems just discussed there are two that deserve the greatest acclaim. In these two papers he examines the so-called "inner" asymptotics for orthogonal systems and the corresponding series expansions. In other words, he discusses questions concerning asymptotic behavior on those curves and intervals on which the polynomials are orthogonalized with respect to some weight function $p(x)$. In this area, where the first classical results are linked with the names of Laplace and Darboux, Szegő not only obtains very general results, far overshadowing anything known previously, but he obtains these results exactly because he examines these questions, considered very difficult, using a simple, one can say elementary, method. The main point of his method is that he squeezes the weight function $p(x)$ between two functions of a very simple structure that have the form $\sqrt{1-x^2}/P(x)$ where $P(x)$ is a polynomial. He shows that these functions may be viewed as majorants and minorants, respectively, from the point of view of the problems which are studied. After reducing the problems to the case of weight functions of this special type, he evaluates explicitly the corresponding expressions, using a theorem of Fejér on positive trigonometric polynomials. Using this method that we have just sketched, in his article, "Über den asymptotischen Ausdruck von Polynomen, die durch eine Orthogonalitätseigenschaft definiert sind" which appeared in the 86th volume of *Math. Annalen*, he gives [inner] asymptotic expressions of orthogonal polynomials for every point x where $p''(x)$ exists. It is known, especially after Haar's dissertation, that with the help of these asymptotic expressions one can reduce questions of convergence and summability of series expansions to certain special cases, e.g., Fourier series. In addition, in another article titled "Über die Entwicklung einer willkürlichen Funktion nach den Polynomen eines Orthogonalsystems" published in the 12th volume of the *Math. Zeitschrift*, Szegő also shows that the same elementary method, without the use of asymptotic expressions of the polynomials, directly gives asymptotics for the partial sums [of orthogonal series] and in this way reduces convergence problems to analogous questions for Fourier series.

I wish to mention another merit of Szegő of a different nature. Namely, the many careful, precise, to-the-point, and, if needed, critical reviews which he wrote for the last two volumes of *Jahrbuch über die Fortschritte der Mathematik*, which dealt with the literature from 1914 to 1918. In my judgement, with these reviews, together with those of other collaborators, he has considerably contributed to raising the quality of the yearbook. This is of permanent value while contacts between scientists of different nations is made difficult by financial and other considerations.

Based on these observations I recommend that the board approve the committee's recommendation.

Szeged, March 7, 1924
Frederick Riesz

Department of Mathematics
University of Wisconsin–Madison
Madison, WI 53706, USA
e-mail: askey@math.wisc.edu

Department of Mathematics
The Ohio State University
Columbus, OH 43210-1174, USA
e-mail: nevai@math.ohio-state.edu
Website: http://www.math.ohio-state.edu/~nevai