Math 366 Homework 3

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We thank to Soonsang Hong for sharing his HW solutions with us.

2.3

22.

a. Given any non zero real number, you can find a real number so that the product of two is one. (This statement is true because every non-zero real number x has a reciprocal $\frac{1}{x}$)

b. There is a real number, whose product with any real number is equal to 1. (This statement is false. Say this real number is x_0 then set $y_0 = \frac{1}{x_0^2 + 100}$. Clearly, $x_0 y_0 = \frac{x_0}{x_0^2 + 100} \neq 1$)

25.

a. The statement is false. Circles b and c are not above triangle d.

b. Negation: $\exists$ a circle x and triangle y such that x is not above y.

27.

a. This statement says that there are a circle and a square such that the circle is above the square and has a same color as the square. (This statement is true. For example, the circle a is above the square j and they have the same color.)

b. Negation: $\forall$ circles x and $\forall$ squares y, x is not above y or x and y do not have the same color.

29.

a. $\forall x \in R$, $\exists y \in R^-$ such that x > y

b. $\exists x \in R \; \forall y \in R^-$, x > y: true (for example x=0)

$\forall x \in R$, $\exists y \in R^-$ such that x > y: true (for any given x, set $y = -|x| - 1$. Clearly, $x > y$ and y is negative)

35.

a. $\exists$ a person x such that $\forall$ people y, x trusts y.

b. $\forall$ people x, $\exists$ a person y such that x does not trust y.

54. These statements are not necessarily equivalent. For example, set $D = R$, P(x)="x is positive" and Q(x)="x is negative". Then $\exists x \in R, (P(x) \wedge Q(x))$ can be written as "There exists a real number which is both positive and negative". This is false. However, $\exists x \in R, P(x) \wedge \exists x \in R, Q(x)$ can be written as "There exists a real number which is negative and there exists a real number which is positive". This is true.

56.

a. These statements have the same truth values for all domains D and predicates P(x) and Q(x).

If the statement $\exists x \in R, (P(x) \vee Q(x))$ is true, then by definition of truth values for $\exists$, the predicate $P(x) \vee Q(x)$ is true for at least one element x in D. Let's call such an element x_0. Then $P(x_0) \vee Q(x_0)$ is true, and so by definition of truth values for $\vee$,at least one of $P(x_0)$ or $Q(x_0)$ is true. In case $P(x_0)$ is true, then the statement $\exists x \in D, P(x)$ is true. In case $Q(x_0)$ is true, then the statement $\exists x \in D, Q(x)$ is true. Since at least one of these cases must occur, the statement $(\exists x \in D, P(x)) \vee (\exists x \in D, Q(x))$ must be true by definition of truth values of $\vee$.

If the statement $(\exists x \in D, P(x)) \vee (\exists x \in D, Q(x))$ is true, then by definition of truth values of $\vee$, at least one of the statements $\exists x \in D, P(x)$ or $\exists x \in D, Q(x)$ must be true. In case $\exists x \in D, P(x)$ is true, then by definition of truth values of $\exists$, there exists an element, say x_1, in D such that $P(x_1)$ is true. Then by definition of $\vee$, $P(x_1) \vee Q(x_1)$ is true, and so by definition of $\exists$, $\exists x \in R, (P(x) \vee Q(x))$ is true. Similarly, in case $\exists x \in D, Q(x)$ is true, then by definition of truth values of $\exists$, there exists an element, say x_2, in D such that $Q(x_2)$ is true. Then by definition of $\vee$, $P(x_2) \vee Q(x_2)$ is true, and so by definition of $\exists$, $\exists x \in R, (P(x) \vee Q(x))$ is true. Since one of the two cases must occur, we can conclude that the statement $\exists x \in R, (P(x) \vee Q(x))$ is true.

2.4

11. Invalid, Converse error

12. Invalid, Inverse error

13. Valid, Universal Modus Ponens

18. Valid, Universal Modus Tollens

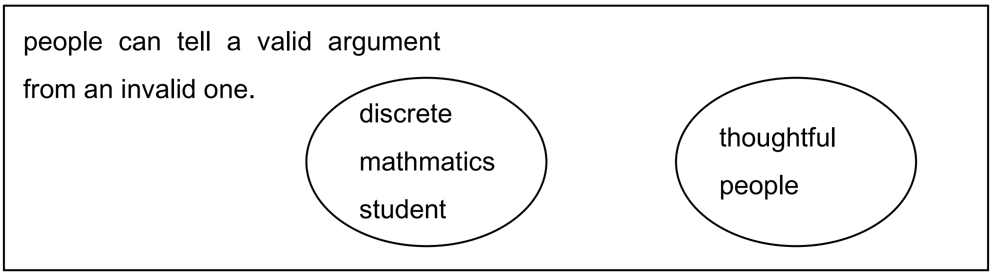
19. $\forall$ x, if x is good car, then x is not cheap.

c. Valid, Universal Modus Tollens

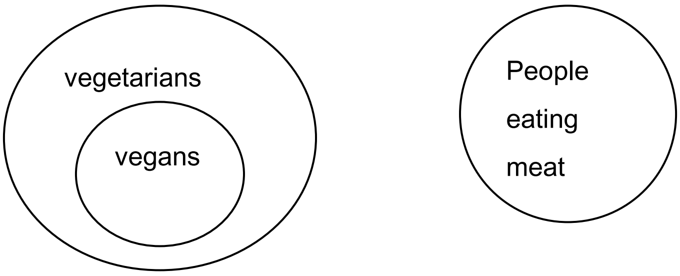
d. Invalid, Inverse error

22. Invalid

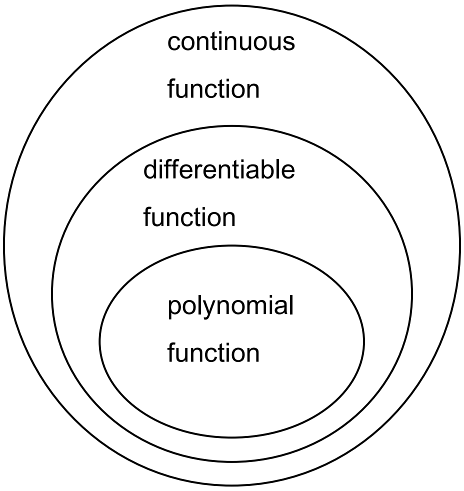
The counter example



24. Valid



26. Valid



29 2. *Contrapositive Form:* If an object is a square, then it is above all the black objects.

3. If an object is above all the black objects, then it is to the right of all the triangles.

1. If an objects is to the right of all the triangles, then it is above all the circles.

$\therefore$ If an object is a square, then it is above all the circles.

3.1

3.

a. Yes, because $4rs = 2(2rs)$ and 2rs is an integer since 2, r and s are integers and product of integers is an integer.

b. Yes, because $6r + 4s^2 + 3 = 2(3r + 2s^2 + 1) + 1$ and $3r + 2s^2 + 1$ is an integer because 3, 2, r, s and 1 are integers and products and sums of integers are integers.

c. Yes, because $r^2 + 2rs + s^2 = (r+s)^2$ and $(r+s)$ is an integer that is greater than or equal to 2 since both r and s are positive integers and thus each is greater than or equal to 1.

5. For example, let m = -1 and n = 1, then m and n are integers and $\frac{1}{m} + \frac{1}{n} = \frac{1}{-1} + \frac{1}{1} = 1 - 1 = 0$, which is an integer.

12. *Counterexample:* Let n = 1, then n is integer and (n – 1)/2 = 0 is even.

18.

n = 1, 1 – 1 + 11 = 11, which is prime.

n = 2, 4 - 2 + 11 = 13, which is prime.

n = 3, 9 - 3 + 11 = 17, which is prime.

n = 4, 16 - 4 + 11 = 23, which is prime.

n = 5, 25 - 5 + 11 = 31, which is prime.

n = 6, 36 - 6 + 11 = 41, which is prime.

n = 7, 49 - 7 + 11 = 53, which is prime.

n = 8, 64 - 8 + 11 = 67, which is prime.

n = 9, 81 - 9 + 11 = 83, which is prime.

n = 10, 100 - 10 + 11 = 101, which is prime.

21. *Theorem:* $\forall$ real numbers x, if x > 1 then $x^2 > x$

Start of proof: Suppose x is any *[particular but arbitrarily chosen]* real number such that $x > 1$. *[We must show that $x^2 > x$]*

28. *Proof:* Suppose n is any odd integer.*[We must show that n^2 is odd].* By definition of odd, n = 2m + 1 for some integer m. By basic algebra, $n^2 = (2m + 1)^2 = 4m^2 + 4m + 1 = 2(2m^2 + 2m) + 1$. Let $k = 2m^2 + 2$. Then k is an integer because sums and products of integers are integers. Thus $n^2 = 2k + 1$, where k is an integer, and so, by definition of odd, n^2 is odd *[as was to be shown].*

42. *Proof:* Suppose m is any even integer and n is any integer.*[we must to show that mn is even].* By definition of even, m = 2k for some integer k. By basic algebra, mn = (2k)n = 2(kn). Let t = kn. Then t is an integer because products of integers are integers. Thus nm = 2t, where t is an integer, and so, by definition of even, nm is even *[as was to be shown].*

3.4

5.

x = 0.565656565656….. 100x = 56.56565656…..

100x – x = 56.56565656…. – 0.5656565656…..

99x = 56

x = 56/99

15. *Proof:* Suppose r and s are rational numbers. By definition of rational, $\mathrm{r} = \frac{\mathrm{a}}{\mathrm{b}}$ and $\mathrm{s} = \frac{\mathrm{c}}{\mathrm{d}}$ for some integers a, b, c and d with $b \neq 0$, $d \neq 0$. Then $\mathrm{r} - \mathrm{s} = \frac{\mathrm{a}}{\mathrm{b}} - \frac{c}{d} = \frac{ad - bc}{bd}$ by substitution and basic algebra (by taking common denominator). Now ad-bc and bd are both integers and bd $\neq$ 0 because products and sums of integers are integers and multiplication of non zero integers is a non zero integer. Thus, r - s is a quotient of integers with a non zero denominator, and so, by definition of rational, r – s is rational.

19. *Proof:* Suppose r and s are any rational numbers. By Exercise 17, $\frac{\mathrm{r}+\mathrm{s}}{2}$ is rational. Let $t = \frac{\mathrm{r}+\mathrm{s}}{2}$. By Exercise 18, $r < t < s$. Thus, there is another rational number t between r and s.

24. *Proof:* Suppose r is any rational number. Then $r^2 = r \cdot r$ is a product of two rational numbers and hence, is rational by Exercise 12 (by solution of exercise 13). Also 2 and 3, which are integers, are rational by exercise 11. Thus both $3r^2$ and $2r$ are rational by the solution to exercise 13 (because they are products of rational numbers), and by the solution to exercise 15, $3r^2 - 2r$ is rational (because difference of two rational numbers is rational). Finally, 4, which is an integer, is rational by exercise 11. So, by Theorem 3.2.2, $3r^2 - 2r + 4 = (3r^2 - 2r) + 4$ is rational. (because it is sum of two rational numbers).