**Math 254.01, Quiz 4** SOLUTIONS

October 29th, 2009

*Instructions:* Show all of your work and justify all steps. Name theorems where appropriate. Correct answers are not worth points without the accompanying work. Write legibly. **No decimal answers will be accepted unless there are decimals in the problem.** You have 20 minutes.

[10 points] Maximize $f(x, y, z) = xyz$ subject to $x + y + z = 30$ and $x, y, z > 0$ (using Lagrange Multipliers).

First, note that $\nabla f(x, y, z) = \langle yz, xz, xy\rangle$; if $g(x, y, z) = x + y + z$, then $\nabla g(x, y, z) = \langle 1, 1, 1\rangle$. So, our system of equations here is

$$yz = \lambda \quad xz = \lambda \quad xy = \lambda \quad x + y + z = 30$$

Now, $x \neq 0$, so $xy = \lambda = xz$ implies $y = z$. Also $z \neq 0$, so $xz = \lambda = yz$ implies $x = y$. Hence $x = y = z$; plugging this in to the constraint yields $x + x + x = 30$, or $x = 10$. So then $y = 10$ and $z = 10$ as well. So, the maximum we can get is $f(10, 10, 10) = 1000$.

[10 points] Let $R = [0,1] \times [0,2]$. Compute

$$\iint\limits_R xye^{x^2y}\, dA$$

Note that

$$\iint\limits_R xye^{x^2y} dA = \int_0^2 \int_0^1 xye^{x^2y}\, dx\, dy$$

For the inner integral, consider the substitution $w = x^2y$. Then $dw = 2xy\, dx$; so, we get

$$\int_0^1 xye^{x^2y}\, dx = \int_0^y \frac{1}{2}e^w\, dw = \left[\frac{1}{2}e^w\right]_{w=0}^y = \frac{1}{2}(e^y - 1)$$

So, we have

$$\iint\limits_R xye^{x^2y}\, dA = \int_0^2 \frac{1}{2}(e^y - 1)\, dy = \left[\frac{e^y - y}{2}\right]_{y=0}^2 = \frac{e^2-2}{2} - \frac{1}{2} = \frac{e^2-3}{2}$$