

## Some useful formulas

### Some trigonometric formulas

$$\sin^2 x = \frac{1 - \cos 2x}{2}, \quad \cos^2 x = \frac{1 + \cos 2x}{2}, \quad \tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

### Some integration formulas

$$\int \sqrt{x^2 + 1} dx = \frac{1}{2} x \sqrt{1 + x^2} + \frac{1}{2} \ln \left( x + \sqrt{1 + x^2} \right) + C$$

$$\int \frac{1}{\sqrt{x^2 + 1}} dx = \ln \left( x + \sqrt{1 + x^2} \right) + C$$

### Conics

*Ellipse/Hyperbola*  $x^2/a^2 \pm y^2/b^2 = 1$

*In the case*  $a > b$ : denote  $c = \sqrt{a^2 - b^2}$  for ellipse and  $c = \sqrt{a^2 + b^2}$  for hyperbola and, for  $a > b$ : Eccentricity:  $e = c/a$  Foci:  $(\pm c, 0)$

*In the case*  $a < b$ : the roles of  $x$  and  $y$  are exchanged in the formulas above.

*Parabola:*  $x^2 = 4py$  Focus  $(0, p)$ .

### Polar coordinates

Element of length:  $ds = \sqrt{dr^2 + r^2 d\theta^2}$

Area bounded by  $r = r(\theta)$ ,  $\theta = \theta_0$ ,  $\theta = \theta_1$  is  $A = \int_{\theta_0}^{\theta_1} \frac{1}{2} r^2 d\theta$

Angle  $\psi$ :  $\tan \psi = \frac{r}{dr/d\theta}$

**Rotation of the rectangular coordinate axes  $Oxy$  by angle  $\theta$  to new coordinate axes  $Ox'y'$ :**  $x = x' \cos \theta - y' \sin \theta$ ,  $y = x' \sin \theta + y' \cos \theta$