Automotive Repair by Number Theory

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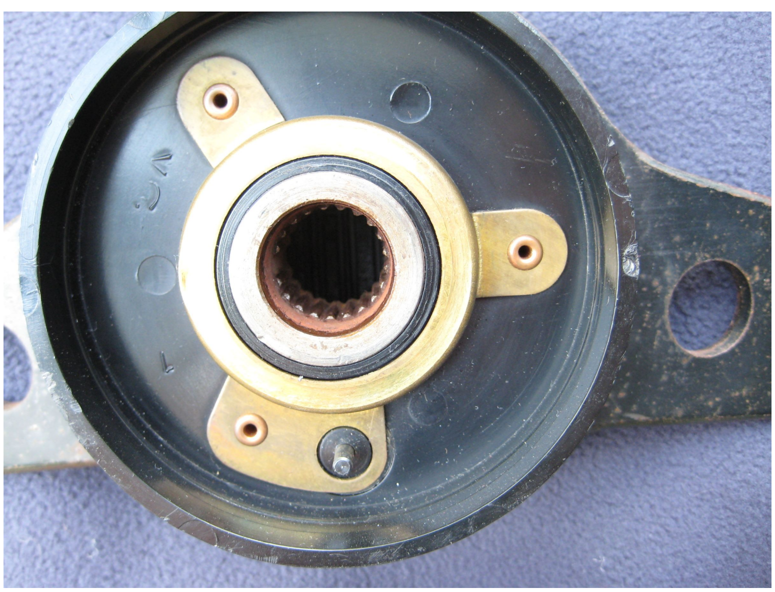
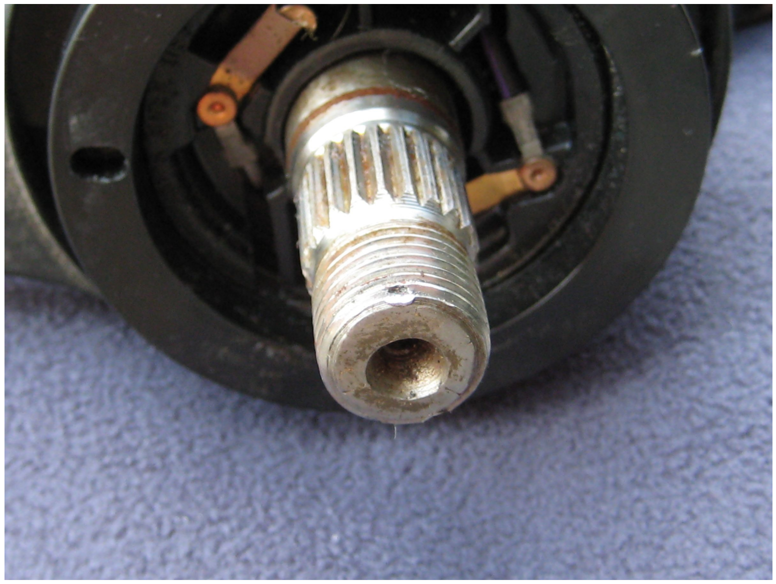
While repairing the ignition switch on a 1981 Fiat Spider, we discovered the following connection between number theory and automotive repair.

In the course of our work on the Spider, we removed the steering wheel and the steering column without marking their original positions. All was fine until it came time to put the parts back together. The steering wheel was no longer centered! The car could drive down the street just fine, but when the car drove straight ahead the steering wheel was off by a rotation of 5 degrees to the right. This would not do!

After some reflecting and calculating, we realized that the problem stemmed from our understanding of the design of the steering wheel/column assembly. To elaborate, the steering column connects the steering wheel to the rest of the car. Here is a picture of the back of the steering wheel:



As you can see, the wheel has 21 notches which match up with another 21 notches on the top of the steering column:



The steering column then attaches to the car with a 17 notch connector:

Hence to center the steering wheel, it is necessary to somehow figure out how to adjust the orientation of both the steering wheel and the steering column.

Question: *How are we to fix this conundrum?*

We'll use number theory of course! Specifically, we want to use facts about the greatest common divisor (GCD) and Diophantine equations.

Question: *What's a Diophantine equation?*

Glad you asked! A *Diophantine equation* is an equation whose unknowns are assumed to be integers. To see the connection between Diophantine equations and this problem, consider the following: We found that there were 21 different orientations (equally spaced) of the steering wheel and 17 different orientations (equally spaced) of the steering column. The upshot is that adjusting the orientation of the wheel by 1 notch causes about a 17 degree shift, while adjusting the orientation of the column by one notch causes about a 21 degree shift—you can see this by dividing 360 by 21 and 17 respectively.

Hence to adjust the wheel so that it will be turned 1 degree to the right from its current position amounts to solving the Diophantine equation

$$17w + 21c = 1$$

where w represents how many notches to the right we adjust the wheel and c represents how many notches to the right we adjust the column. Now we must point out something important:

A Diophantine equation of the form $ax + by = g$ has a solution if and only if the GCD of a and b divides g.

Since the GCD of 17 and 21 is 1, we know our equation has a solution. How do we find the solution? It turns out that there is an old trick which is related to the Euclidean algorithm. Essentially, in the process of computing the GCD, we are able to find a solution to the equation. In this case we find that $w = 5$ and $c = 4$:

$$(17)(5) - (21)(4) = 1$$

So if the wheel is off by 5 degrees to the right, we want to turn the wheel 5 degrees to the left. So multiply the above equation by -5,

$$(17)(5)(-5) - (21)(4)(-5) = (1)(-5)$$

and write:

$$(17)(-25) + (21)(20) = -5$$

So this means the wheel needs to be turned 25 notches to the left, and the column needs to be adjusted 20 notches to the right. However, there are only 21 different orientations of the wheel, and so we really only need to adjust the wheel 4 notches to the left (25 is congruent to 4 modulo 21). On the other hand, there are only 17 different orientations of the column, and so we really only need to adjust the column 3 notches to the right (20 is congruent to 3 modulo 17).

Summing up, we adjusted the wheel 4 notches to the left and adjust the column 3 notches to the right and got the steering wheel looking pretty centered. OK—we think we are done with that question. However, there are always more questions! In particular:

Question: *Why did the Fiat engineers choose the numbers* 17 *and* 21*? Were they chosen purposely, or were they an example of questionable engineering?*

Well, apparently the Fiat engineers knew that they would be wanting to adjust the steering wheel in 1 degree increments. Since the square-root of 360 is about 19, they would want two integers close to 19 such that they have no common factors other than 1. Since 18 and 20 won't work, they used 17 and 21. Number theory to the rescue again!

If you think this sort of thing is interesting, we encourage you to check out: *Number Theory Through Inquiry* by David C. Marshall, Edward Odell & Michael Starbird.