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Calculators in Mathematics Teaching and Learning
Past, Present, and Future

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THE 1986 National Assessment of Educational Progress (NAEP) revealed that 21 percent of the middle school students in the study and 26 percent of the senior high school students in the study attended schools that had calculators available in the mathematics classroom (Dossey et al. 1988, p. 79). In the 1992 NAEP study, these percents had risen to 81 percent and 92 percent, respectively (Dossey and Mullis 1997, p. 26). In 1990, NAEP results showed that 33 percent of eighth graders in public schools were allowed to use calculators for mathematics tests. By 1996, this percent had risen to 70 percent of all eighth graders (Shaughnessy, Nelson, and Norris 1998, pp. 93–95, 191), and nearly 60 percent of eighth graders were using calculators in their mathematics classes on a daily basis! Seldom in the history of mathematics education has such a rapid change been made with such significant consequences. In this article we would like to step out of the rushing stream of this technology-induced change, pause to reflect on our experiences of the past twenty-five years, and describe what we see on the horizon for calculators in mathematics education in the twenty-first century.

We begin with a brief history of calculator development and give some lessons learned about using calculators in our work dating back to the 1970s. A statement of our position on the appropriate use of calculator technology is presented along with a discussion of some of the controversial issues that have arisen as a consequence of the use of calculators. We describe the importance of a balanced approach to the teaching and learning of mathematics that uses both technology and paper-and-pencil techniques. Some research evidence about calculator use is given. Finally, we discuss the status of the use of calculators in the teaching and learning of mathematics in the rest of the world, describe some recent advances in calculator technology, and speculate about the future impact of these advances.

A BRIEF HISTORY

According to Ball (1997), handheld electronic calculators were first introduced to the world by Canon, Inc., in this 14 April 1970 press release from Japan:

> Canon Inc., in close collaboration with Texas Instruments Inc. of the United States, has successfully developed the world's first "pocketable" battery-driven electronic print-out calculator with full large-scale integrated circuitry.

In 1972, Hewlett-Packard introduced the remarkable HP-35, the first "scientific" calculator that evaluated the values of transcendental functions such as log 3, sin 3, and so on. The last slide rule was manufactured in the United States in 1975! In 1986, Casio of Japan introduced the first so-called graphing calculator with powerful built-in, computer-like graphing software. In 1996 Texas Instruments introduced the TI-92, the first calculator that contained an easy-to-use computer algebra system (CAS) *and* a version of Cabri computer interactive geometry (Waits and Demana 1996). Recently, both Texas Instruments and Casio introduced flash ROM calculators, which have many positive implications for the future. Flash technology will enable many kinds of useful computer programs to run on calculators as well as provide easy calculator software upgrades electronically. This feature alone could revolutionize the applicability of calculators in the twenty-first century.

WHAT WE LEARNED ABOUT USING CALCULATORS IN MATHEMATICS TEACHING

After twenty-five years of using handheld calculators, we have learned some fundamental principles about the use of calculators in the teaching and learning of mathematics (Waits and Demana 1994).

We have learned important lessons about change. Arguably the most important thing we learned has to do with desktop computers and why they had very little direct impact on the teaching and learning of school mathematics. We tried using desktop computers in the 1980s in our early projects in which we used our own computer graphing software (i.e., Master Grapher [Waits and Demana 1987]) to enhance the understanding of precalculus and calculus. Whereas teachers became very excited about the possibilities, most students in most schools, we discovered, had very limited, if any, access either to desktop computers or to mathematics computer software in their mathematics classrooms. We found that our ideas were not being used. Our work had very little impact on classroom instruction.

When graphing calculators were introduced, we saw an obvious opportunity because they were very inexpensive, handheld (fit in a shirt pocket), and very computer-like. We immediately began to instruct the teachers in our projects in the use of graphing calculators. The rest is history, as they say. Graphing calculators soon became very popular in many countries, including the United States. The reasons were obvious: every classroom could be turned into a computer lab, and every student could own his or her own inexpensive personal computer with built-in mathematics software (Demana and Waits 1992). We note that the same dynamics are true today for CAS. For example, graphing calculators now have computer algebra systems almost as powerful as personal computer–based software like Mathematica or Maple.

What we have learned does not imply that desktop computers are not important in education! All the software and functionality available today on advanced calculators first appeared on desktop computers. In many ways the calculator borrows what has been proved effective on the computer and makes it accessible to many more students. The lesson we learned is that *change can occur if we put the potential for change in the hands of everyone.* The handheld calculator does precisely this for the mathematics teacher and student. The result is clearly demonstrated by taking a cursory look at the data presented in our opening paragraph.

The second most important thing we learned about change has to do with effective professional development in the use of technology. The adoption and use of technology requires additional teacher in-service training that addresses conceptual and pedagogical issues. However, our early professional development methods consisted largely of demonstrating the use of technology to the large groups of teachers we brought to Ohio State University. This form of teacher in-service training was not good enough. We cannot expect teachers to make fundamental change in their teaching without adequate, ongoing support. Teachers consistently request intensive start-up assistance and regular follow-up activities. But the greater lesson we learned is that teaching in the grades K–12 arena is a profession whose constraints are so complex and abundant that teaching practice is very difficult to change from the outside. Change has to come from within the teaching profession and be supported both from within and from without.

Changing practice is full of local issues that must be dealt with by teachers at that level. Our early top-down, one-dimensional model of professional development simply had no hope of producing the change we wanted to extend to all schools and all mathematics teachers. The best thing we ever did was to turn the professional development activities of our projects over to practicing teachers who had succeeded in embedding the appropriate use of calculators into their own practices.

We have learned that on a large scale, *it takes practiced teachers to change the practice of teachers.* The Teachers Teaching with Technology (T^3) program that we founded in 1985–86 is an example of such a professional development program. The T^3 program has consistently tried to embody the tried-and-true principles of effective professional development, many of which we learned the hard way in our projects. (For an excellent analysis of effective professional development experiences, see Loucks-Horsley et al. [1998].)

T^3 offers intensive teacher education institutes, and the regional and annual T^3 meetings afford teachers opportunities to obtain ongoing professional development. Practicing teachers in the T^3 institutes model appropriate calculator use in teaching specific mathematics and science topics. The T^3 organization also supports its institutes by providing an extensive Web page filled with resources for teachers using calculators in their classrooms as well as areas for discussion, where teachers can get help and share ideas (www.t3ww.org/t3).

We have learned that calculators cause changes in the mathematics that we teach. These changes often can be very dramatic, as we learned from our personal teaching experiences before and after calculators were available. For example, some paper-and-pencil applications have simply become obsolete, as illustrated by the following examples:

- Complicated arithmetic computation: Compare computing
$$\frac{1789}{1.0725}$$
by paper-and-pencil long division with computing the quotient using a simple four-function calculator.
- Interpolation using transcendental-function tables: Compare computing $1250(1.04125)^{12}$ using precalculator, paper-and-pencil logarithmic interpolation with computing the product using an inexpensive scientific calculator.
- Accurate graphing of complicated functions: Compare graphing
$$f(x) = \frac{x^3 - 17x + 7}{x^2 + 1}$$
using traditional paper-and-pencil calculus methods with graphing the function using a graphing calculator (see fig. 5.1). In the past the traditional methods for "graphing" included finding the derivative, $f'(x)$, and solving the equation $f'(x) = 0$ by paper-and-pencil methods only. And only contrived graphing problems with accessible paper-and-pencil solutions would be given. Now students can graph far more functions more accurately than before. Also students can

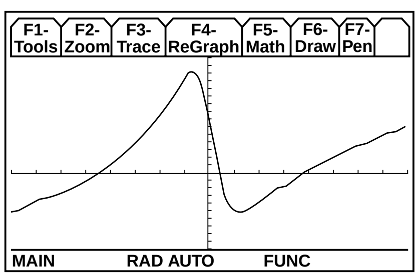

Fig. 5.1. An accurate graph of the function
$$f(x) = \frac{x^3 - 17x + 7}{x^2 + 1}$$

use traditional calculus methods to *confirm analytically* that the graph they see is accurate.

- Complicated integrations: Compare computing the value of the definite integral

$$\int_0^{\frac{\pi}{3}} x^2 \sin(x)\,dx$$

using paper-and-pencil methods with computing the value using a state-of-the-art CAS calculator (see fig. 5.2).

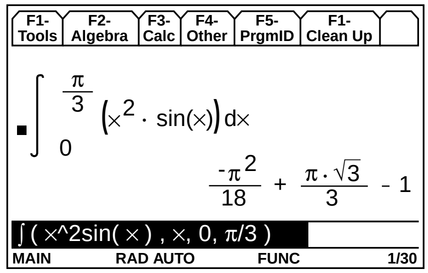

Fig. 5.2. Using the TI-89 to compute the *exact* value of

$$\int_0^{\frac{\pi}{3}} x^2 \sin(x)\,dx$$

- Solving complicated equations: Compare finding the real and complex solutions to the simple cubic polynomial equation $3x^3 + 2x^2 - 7x + 9 = 0$ by paper-and-pencil methods (go ahead and try!) with using a calculator-based graphical or numerical method.

We have learned that before calculators, we often asked students to solve only contrived problems. The students, therefore, learned methods that, at least in their minds, were really applicable only in contrived contexts. Calculators allow students to apply more-general types of solution processes even to problems that have no exact solution or to problems that cannot be solved by traditional paper-and-pencil methods alone, as illustrated in the next examples.

- Problems not solvable by paper-and-pencil methods taught in schools: Compare computing the definite integral

$$\int_1^2 \frac{\sin(x)}{x}\,dx$$

with paper and pencil to computing the solution using a calculator with integration functionality. Can you find the indefinite integral as an elementary function (Demana and Waits 1994)?

- Methods too cumbersome before calculators: The parametric graphing utility on most graphing calculators makes possible mathematical modeling and simulation to illustrate and solve problems that were impossible with paper and pencil alone.

Clearly we can solve many more problems using calculators! They facilitate problem solving. In general, when it comes to mathematics, we have learned that the visionary statements made by the eminent mathematician Henry Pollak are very true (Pollak 1986, pp. 347–48). To paraphrase, he said that because of technology—

- some mathematics becomes less important (like many paper-and-pencil arithmetic and symbol-manipulation techniques);
- some mathematics becomes more important (like discrete mathematics, data analysis, parametric representations, and nonlinear mathematics);
- some new mathematics becomes possible (like fractal geometry).

We have learned that calculators cause changes in the way we teach and in the way students learn. Before computers and calculators, it was necessary for students to spend time mastering and becoming proficient in the use of paper-and-pencil computational and manipulative techniques. Today much of this time can be spent on developing deeper conceptual understanding and valuable critical-thinking and problem-solving skills. We have found the following to be true:

- Calculators reduce the drudgery of applying arithmetic and algebraic procedures when those procedures are not the focus of the lesson. They provide better ways to compute and manipulate symbols. For example, if the problem is to find the area of a region bounded by the graphs of two functions, then the essential challenge for the student is to understand that a definite integral is needed, determine the limits of integration, and set up the specific definite integral. Finally, the student needs to determine whether the answer obtained makes sense in the problem situation. All these tasks require serious thinking and thorough understanding. The actual computation of the integral is often best done with (or feasible only with) calculator or computer technology.
- Calculators with computer interactive geometry allow for investigations that lead to a much better understanding of geometry (Laborde 1999; Vonder Embse and Engebretsen 1996).
- Calculators help students see that mathematics has value. Students using calculators find mathematics more interesting *and* exciting. Texas Instruments first introduced a handheld calculator-based-laboratory (CBL) device in 1994 that connects to the link port of graphing calculators. This device allows students to make precise measurements of many scientific phenomena and store the measurements in their calculators for mathematical analysis. Thus, more than any other classroom innovation in the past, calculator-based laboratories have connected school mathematics to the real-world phenomena around the student. The excitement and interest in both mathematics and science generated by these real-world connections is impressive (Bruneningsen and Krawiec 1998).
- Calculators make possible a “linked multiple-representation” approach to instruction. A graphing calculator makes graphical and numerical representations practical learning strategies.

- Before calculators we studied calculus (applications of the derivative) to learn *how to obtain accurate graphs.* Today we use accurate graphs produced by a graphing calculator to help us study the concepts of calculus.

WHY THE CONTROVERSY ABOUT USING CALCULATORS IN MATHEMATICS EDUCATION?

Controversy is associated with using technology in the teaching and learning of mathematics. It is human nature not to want to change. It is comfortable to teach in the way we were taught. One of the great problems we face in mathematics education is communicating the real nature and value of mathematics. Before the publication of the *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM] 1989) and before the extensive use of calculators in mathematics classrooms, most students viewed mathematics as a bag of tricks and rules to memorize for computing or solving something. All too many students still do. Personal experience and evidence from the 1986 NAEP (Dossey et al. 1988, p. 102) strongly support this observation. Students also think of mathematics as tedious, boring work, particularly when they remember only the endless drill exercises—the "do it until it hurts" kind. We must communicate the true nature of mathematics and build a case that the appropriate use of technology will enhance the teaching and learning of mathematics. If the true nature of mathematics is understood, then the use of technology in the learning of mathematics will be seen as natural enhancements and extensions.

Paper-and-pencil arithmetic and algebraic symbol-manipulation procedures were very important in the past because they were the only procedures available for computing and solving. Today, teachers must examine on a case-by-case basis which paper-and-pencil arithmetic and algebraic manipulation procedures should still be emphasized in the curriculum. It will become clear that many techniques we teach are still emphasized in the curriculum only because they were the only methods possible in the past. We must distinguish between applying mathematics algorithms and doing real mathematics (Ralston 1999).

What does the research tell us?

We have strong evidence from careful research studies to support the use of technology in the teaching and learning of mathematics. A recent comprehensive listing and analysis of calculator research has been completed by Dunham (in press). One of the most compelling arguments for the use of calculators in mathematics teaching and learning is the meta-analysis of eighty-eight studies on the use of calculators that was conducted by Hembree and Dessart (1992). Only one of these studies reported negatively about calculator use. We cite one of the conclusions drawn from Hembree and Dessart's analysis.

> The preponderance of research evidence supports the fact that calculator use for instruction and testing enhances learning and the performance of arithmetical concepts and skills, problem solving, and attitudes of students. Further research should dwell on the best ways to implement and integrate the calculator into the mathematics curriculum. (P. 30)

The studies reviewed by Hembree and Dessart typically were conducted in classrooms in which the students were taught the traditional paper-and-pencil skills while or before they used calculators. In a longitudinal study done between 1986 and 1992 in Great Britain, the children were never taught the traditional paper-and-pencil algorithms for arithmetic, but over the years the children used calculators and successfully invented their own paper-and-pencil processes for the arithmetic operations (Shuard 1992). One of the interesting, albeit ambiguous, results of the Third International Mathematics and Science Study (TIMSS) was that internationally, on all the tests at the advanced level, students who reported using calculators in their daily coursework performed well above those who rarely or never used them (TIMSS 1998). In general, it is fair to say that the research indicates that the use of calculators does not degrade the basic skills of students. In the review conducted by Hembree and Dessart (1992), calculators did appear to have positive effects on students' problem-solving abilities and attitudes toward mathematics.

How can we make the best use of this technology?

If we want to see both the basic skills and the problem-solving skills of our students improve in contexts that allow the regular use of calculators, then we must continue to develop methods that we might agree to call "appropriate uses," not only for calculators but for paper-and-pencil techniques as well. In this regard, we have come to the conclusion that a balanced approach of paper-and-pencil techniques and technology in the teaching and learning of mathematics is essential. We need to communicate that some traditional arithmetic and algebraic skills are still very important. Indeed, we believe they will be even more important in the future as we move to more computer-intensive learning environments. For example, without using calculators, students need to be able to multiply quickly two numbers, one of which contains a single digit or is a power of 10, and be able to divide a number by a single-digit divisor or a power of 10. Computing the products and quotients of other numbers can be left to the calculator. We want students to be able to explain why $16! = 2^{15} \cdot 3^{6} \cdot 5^{3} \cdot 7^{2} \cdot 11 \cdot 13$, but we would not ask them to use paper and pencil to obtain the factorization.

Brolin and Björk (1992) and Wheatley and Shumway (1992) support our position. In a national project in Sweden, training in arithmetic algorithms in grades 4–6 was reduced in favor of extending the use of calculators for solving more-complicated problems (Brolin and Bjork 1992). For example, it was considered sufficient if students could divide by single digits. The results of the project showed that students who used the calculator did not lose important basic skills in algorithmic calculations. Wheatley and Shumway (1992) state that "it would be much more important that students *know when to subtract* than that they be able to use a prescribed and complex subtraction algorithm efficiently" (p. 2).

Balance means the appropriate use of paper-and-pencil *and* calculator techniques on a regular basis. Used properly, paper and pencil and calculators can complement each other. It is important to know how to estimate an answer before doing a computation using either a calculator or paper and pencil. It is important that students have enough number sense to recognize when answers are correct and that they know methods of checking answers without doing the problem over. And it is important for students to understand at least on an intuitive level why procedures work and when they are applicable. Balance does not mean that we quit teaching such skills as long division or factoring. No one should simply dictate that. However, it does mean that our objectives for mastery and understanding shift from speedy paper-and-pencil computation in division and factoring problems to making sense of the operations and their proper use.

Time must be provided in the curriculum for *appropriate* practice of these needed skills. One method teachers use to achieve a good balance is to have students routinely employ each of the following three strategies:

1. Solve problems using paper and pencil and then *support* the results using technology
2. Solve problems using technology and then *confirm* the results using paper-and-pencil techniques
3. Solve problems for which they choose whether it is most appropriate to use paper-and-pencil techniques, calculator techniques, or a combination of both

These approaches help students understand the proper use of technology.

Another approach to achieving balance is to use manipulatives and paper-and-pencil techniques during the initial concept development and use calculators in the extension and generalizing phases. There is a vast difference between achieving precision in a paper-and-pencil procedure and explaining why the paper-and-pencil procedure works. Solving systems of two linear equations in two unknowns provides a good illustration of this difference. We could expect students to solve several systems using paper-and-pencil substitution. Then using this knowledge, students could extend their understanding by solving the same systems, step-by-step, with a computer algebra system. Transferring the process to a computer algebra system requires that students understand the process. Finally, with the computer algebra system we can challenge the students to solve a general system of two linear equations in two unknowns to obtain their own version of Cramer's rule.

Whatever our methods may be for achieving the balance we are calling for, we must *not* back off on the full, regular, and integrated use of available technology, including graphing calculators with computer algebra, computer interactive geometry, and computer software microworlds, discussed below, in *all* school mathematics classes.

In order to achieve the balance we are calling for, assessment needs to be given a more prominent role in professional development activities. High school teachers being introduced to calculators with CAS often remark that if they allowed such calculators in their classrooms, they would have to change all their tests and quizzes! Professional development could address such concerns by discussing when to test with technology and when to test without it. It is all right to give some tests without technology. It is *not* all right to give all tests without technology because doing so makes technology seem unimportant and an add-on to the curriculum. The use of technology must be truly integrated into the fabric of classroom practice. Indeed, new textbooks are needed that integrate technology into the fabric of the curriculum.

Whenever students use calculator technology on a regular basis, we run the risk that some will develop misconceptions because of the limitations of the calculators or inappropriate use. For example, many instances of inappropriate calculator use stem from a lack of understanding of how a calculator draws a graph. (It samples only a *discrete* number of function values and connects the associated points.) Errors can obviously occur when a discrete device like a graphing calculator is used to model continuous functions (Demana and Waits 1988). Designers of professional development programs must understand that with every advance in calculator technology, teachers must not only be updated but also be made keenly aware of the limitations of the technology.

Teachers' fears about technology need to be understood and addressed. New CAS calculators can perform most of the traditional algebra and calculus symbolic manipulations. Unfortunately, most classroom teachers today spend the majority of their time on the very same manipulations using paper-and-pencil techniques, some of which will soon be obsolete. CAS tools do the manipulations faster and more accurately than any teacher or student. Student use of these new tools will require many changes. Curricula will change. Tests will change. Expectations will change. Teachers who are not willing to change will indeed fear technology.

At a higher level, to achieve the called-for balance, new state tests are needed that acknowledge technology. Documents like the *Principles and Standards for School Mathematics* (NCTM 2000) should be used as a catalyst for the discussion of such issues. New, generally applicable pedagogical approaches need to be developed, tested, and disseminated. For example, some Austrians have developed little-known but powerful strategies for using computer algebra systems in algebra and calculus. These strategies are known as the black-box–white-box and the scaffolding principles (Heugl, Klinger, and Lechner 1996). Consider teaching long division of polynomials. In the white-box phase no calculators would be used except perhaps to check results. Paper-and-pencil procedures would be developed that illustrate the division algorithm and why it works. Later in the year, when division is needed in a problem, students would be allowed to use a calculator for the computation (black-box phase).

RECENT ADVANCES IN CALCULATOR TECHNOLOGY

Perhaps the single most significant advance in calculator technology that has huge ramifications for the future of calculators in mathematics classrooms has been the invention of "flash ROM."

What does flash ROM in a calculator do?

Flash ROM is a new type of calculator memory first introduced by Texas Instruments in 1998. Until recently calculators had only two types of distinct memory, ROM and RAM. ROM, or read only memory, can be programmed only once and never changed. All the built-in functionality that comes with a calculator is stored in ROM. ROM is relatively inexpensive, so the amount of ROM used in calculators has increased over the years as more and more functionality has been included. If a calculator had only ROM memory, it would not be possible to enter numbers, store values into variables, or even graph a given function. For these operations, the calculator needs RAM, or random access memory, which allows new information to be stored.

RAM can be rewritten an unlimited number of times. It is used as scratch space during calculations and also as a place to store information such as equations, lists, programs, and so on. RAM has the drawback that it requires more power to operate than ROM, an important consideration for low-power, battery-operated devices like calculators. Also, RAM has the drawback of being relatively expensive. It is usually the second most expensive part of a calculator, after the display. Despite the drop in prices over the last few years for computer RAM, calculator RAM prices have not dropped as fast because calculators use a different type of RAM. To keep the price of calculators low, the amount of RAM in calculators has been restricted. Flash ROM combines the benefits of both RAM and ROM in that it is ROM but it can be rewritten like RAM, although it is currently limited to about one hundred thousand rewrites.

Flash ROM supplies much more memory in a calculator. Already, flash calculators can have six to ten times the amount of user memory found on non-flash graphing calculators. Flash ROM allows calculators to be upgraded electronically. A new version of the built-in mathematical software, or base code, can be downloaded to the calculator, replacing the previous version. Students will be able to upgrade their calculators and add the latest features without buying a new calculator. Also, calculator companies will be able to distribute maintenance upgrades that improve the underlying system without replacing the calculator itself. This feature is very important to teachers and parents for economic reasons, since it will make calculator "boxes" last longer.

Perhaps the most significant implication of flash ROM is that it enables calculator software applications, also called *flash applications*, which will allow the calculators of the future to become small computer platforms for software applications!

What are flash applications?

Flash applications are software programs that run on a calculator. They can do more than user programs developed in the calculator's program editor because they are written in more-powerful software languages (C and assembly language) that tap into more of the underlying calculator system. Flash applications can also be faster than user programs for the same reason. Flash applications provide a way of adding on to the built-in functionality, or base code, with additional software that is similar in construction. Like the base code, flash applications are stored in flash ROM and remain there while running. Therefore, they do not take up valuable RAM space the way user programs do, they stay on the calculator unless they are deliberately deleted, and they can't be accidentally removed by resetting RAM or if the calculator's batteries die.

Flash applications can dramatically change the functionality of a calculator, since they are able to control what is displayed on the calculator screen down to the level of individual pixels. Flash applications are not limited to displaying the menus, home screen, tables, and graphs of a standard graphing calculator but can also display pictures, animations, icons, new types of menus, and so on. Lessons and activities that used to be delivered as worksheets or textbook exercises can be illustrated, animated, and electronically linked to the calculator's computational features.

For example, Puzzle Tanks (Sunburst Communications1999) is a flash application for developing mathematical problem-solving skills. Puzzle Tanks animates the standard problem of obtaining a given quantity of liquid using tanks of different fixed sizes. It shows the tanks and liquid levels on the calculator display and updates them interactively as the student enters estimates (fig. 5.3). The game involves four levels of play, each level increasing with difficulty. Notice the remarkable change in the look of the traditional graphing calculator screen due to the flash application. Anyone familiar with the wide array of educational software available today can see from this example the

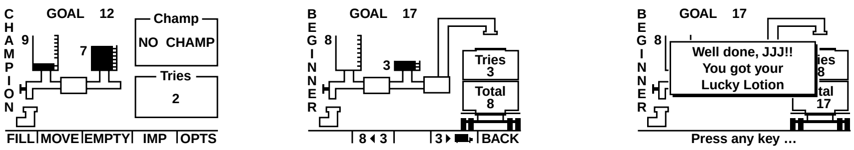

Fig. 5.3. TI-73 screen dumps from Puzzle Tanks

educational threshold that calculators are about to cross because of flash ROM. (For other examples, see www.ti.com/calc/flash/73apps .htm#pt.)

THE MARRIAGE OF CALCULATORS AND COMPUTERS AND OTHER PREDICTIONS

No one can know with any certainty what tomorrow will bring in the area of classroom calculator technology. However, we believe our crystal ball is clear enough to make a few conjectures about the immediate horizon:

• New flash ROM calculators will hasten the current "engagement" of calculators and computers into a real marriage. Flash calculators will quickly become viewed as pocket, or handheld, computers because most computer software packages now available on desktop personal computers will be adaptable to run on tomorrow's calculators. We believe this will have the effect of tremendously accelerating the use of "computer" software by all students in schools. Just imagine the "microworlds" for teaching mathematics described by visionaries in the 1980s, and even those envisioned today, implemented on inexpensive flash calculators (see, e.g., James Kaput's SimCalc Web page, www.simcalc.umassd.edu/)! Imagine spreadsheets, three-dimensional geometry, Logo, and other powerful mathematical tools running on the calculators of tomorrow! A note of caution is in order, however. As calculators and computers assume more-prominent roles in education, integration among software applications and between software and curricula is essential for the wide-scale adoption of technology in schools (Bork 1995). Roschelle et al. (1998) point out the problem in the abstract of their paper:

> Technology-rich learning environments can accelerate and enhance core curriculum reform in science and mathematics by enabling more diverse students to learn more complex concepts with deeper understanding at a younger age. Unfortunately, today's technology research and development efforts result not in a richly integrated environment, but rather with a fragmentary collection of incompatible software application islands.

• The Internet is causing profound changes in our society and has allowed anyone to access the network using a personal computer. We believe the same changes will soon occur with networked calculators, linking them to one another, to computers, and to the Internet. This networking will have a profound effect on the classrooms of tomorrow similar to that experienced when graphing calculators brought the power of computer visualization to thousands of students who had little or no access to computers.

• We are also likely to see textbook publishers move to integrate more calculator-driven computer software into their lessons. Textbooks might even become thinner!

• The marriage of calculators and computers will allow us to resolve some of the intractable equity issues of our educational system. We predict that the inexpensive flash calculator will become the personal computer for *every* mathematics and science student and students in other disciplines, as well.

• Looking to the future, we cannot fail to note that school mathematics and science standards should be a catalyst for a discussion of the fundamental issues raised in this article. We note also that standardized tests *must* change to reflect the advances technology has made and will continue to make in the curriculum. Used unwisely, standardized tests can have a very detrimental effect on teachers' and textbook publishers' willingness to make needed changes and tackle the hard calculator issues. It will be no different in the twenty-first century unless we as a society allow our old prejudices to be put on the table for discussion.

As we conclude this article, we would be remiss not to point out a trend toward globalization that has been happening for a long time but with little fanfare. Because of the information technologies of the late twentieth century, we believe, this trend has just entered the steep part of an exponential curve of change. We all know that because of technology the world is becoming a smaller place. Innovations no longer remain dormant in isolated regions of the world. The use of graphing calculators in school mathematics is rapidly increasing worldwide. For example, they are used extensively in France, Germany, Scotland, Austria, Sweden, Denmark, Holland, Australia, Portugal, and Canada. There are now organizations like T^3 in twenty-one countries worldwide, including Japan. National tests in Sweden, Denmark, Portugal, and France *require* graphing calculators. Many countries have centralized provincial curricula and tests and require graphing calculators on these tests. In France, students are allowed to use CAS calculators on the final National Lycée exams used for university entrance. (And French high school students achieved the top score on the advanced mathematics part of the recent TIMSS study.) With the capabilities already visible on the horizon of tomorrow's calculators and with calculators' intimate connection with the other rapidly developing computer and software technologies, the impact of calculator and computer technologies on the classrooms of tomorrow will become an international issue. These classroom technologies will very much serve as a catalyst, perhaps even more than studies like TIMSS, for bringing mathematics curricula around the world closer together.

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