The Evolution of Instructional Use of Hand Held Technology. What we wanted? What we got! *

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***** To appear in the Proceedings of the October, 1997 Technology Transitions Calculus Conference.

Introduction

After a little more than two decades of numerous pioneering efforts, we believe that we can safely say that the use of hand held technology has forever changed the way calculus and courses prior to calculus are taught, and forever changed the way students learn this mathematics. Our philosophy about using technology in instruction developed through the experience gained in our work in the Calculator and Computer Precalculus (C^2PC) project (Waits & Demana, 1994) that started in 1985. The C^2PC project extended the work of Demana, Joan Leitzel, A. Osborne, and J. Crosswhite in the Transitions to College Mathematics project that started at The Ohio State University (OSU) in 1980 (Demana & Leitzel, 1988). The Transitions project was expanded to include middle school mathematics in 1983 and called the Approaching Algebra Numerically (AAN) project (Comstock & Demana, 1987). The Transitions, AAN, and C^2PC projects grew out of the OSU effort to reform the college remedial mathematics curriculum that began in 1974 and required the use of four-function calculators by all students (Waits & Leitzel, 1976).

Soon after the C^2PC project started, the Sloan Conference was held at Tulane University in January, 1986, and sparked the calculus reform movement in the U.S. (MAA, 1986). This movement was fueled by the National Science Foundation as it provided millions of dollars in grants for calculus reform shortly after the national conference “Calculus for a New Century” was held in Washington, D.C. in October, 1987 (MAA, 1987) which we attended.

In this paper we will give an overview of the development of graphing calculators, primarily focusing on the developments by Texas Instruments, Inc. We will describe how we think the use of hand held technology has changed the teaching and learning of mathematics. Lessons we have learned from this revolution, and suggestions about how the lessons can be used to guide our efforts in the future are given.

Background

At about the time the Texas Instruments TI-81 and the Hewlett-Packard’s HP-48SX graphing calculators were launched, several of us attending this conference took a stab at making comparisons and recommendations for graphing calculators (Demana, Dick, Harvey, Kenelly, Musser, & Waits, 1990). In this paper we pointed out that scientific calculators had moved from exotic and expensive tools for professionals to commonplace tools when the cost became low enough that high school students could afford them, and school systems could supply them like textbooks. Scientific calculators had established themselves as important tools for secondary mathematics classrooms in less than 10 years, and we predicted that graphing calculators would become far more useful than ordinary scientific calculators.

The *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) had just appeared and declared that "scientific calculators with graphing capabilities should be made available to all secondary students at all times." Many colleges and universities were experimenting with the use of graphing calculators in calculus and precalculus at this time.

We compared the notational display, computational features, graphing features, programming features, and computer algebra features of eight graphing calculators: Casio's *fx-7000G, fx-7500G, fx-8000G,* and *fx-8500G;* Hewlett-Packard's *HP-28S* and *HP-48SX;* Sharp's *EL-5200;* and Texas Instrument's *TI-81*. For high school and college programs that do not require calculus we recommended the TI-81, and for college calculus the *HP-48SX***.** We then made the following list of 16 recommendations for a "perfect graphing calculator."

1. A screen that is as large as possible and that has high resolution (e.g., 300 by 300 pixels on a 3" by 3" screen);
2. User-friendly functionality, and notational display in forms that are used most often at present;
3. The capability to easily input lists of functions and to graph associated algebraic combinations of them;
4. The fast, dynamic graphing of multiple functions;
5. The fast, dynamic graphing of relations, and parametric and polar equations without programming;
6. An automatic zoom feature that has an "on-screen" zoom-in rectangle;
7. Dynamic scrolling of the screen, even when multiple graphs are overlaid;
8. Fast and robust root, intersection, extremum, and inflection point finders;
9. A choice of plotting modes and the capability to shade graphs;
10. Automatic read-out of locations of discontinuities;
11. High-quality, low-cost printer, PC, and large-screen projection devices and related communication interfaces;
12. Options to graph either with or without visual continuity;
13. Easy-to-use symbolic manipulation, including algebraic and calculus manipulations (integration and differentiation);
14. Multiple representations (side-by-side display of tabular-numeric, symbolic, and graphic representations);
15. Unlimited program locations, and at least 512 K of RAM program memory;

16. All of the above at a cost of about $50.

We certainly have obtained the items in 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, and 15. The TI-92 has 239 by 103 pixels and a 3.5" by 2" screen – not quite item 1 but close. We don't have 10, and we certainly haven't achieved the price in item 16. However, the two of us naively stated in 1986 that we would like a scientific calculator with an attached grapher for $35. We note that $35 at 4% inflation for 11 years is about $54, just about the current grant price of a TI-82 or 83. So maybe we were not so naive!

Evolution of Calculators

Ten years ago the gap between desk top computers and calculators was quite large. Computers were powerful, expensive, and ran sophisticated software. Calculators were inexpensive and did only elementary numerical computations. Electronic calculators are now over 25 years old while desk top computers are only about 20 years old. The first electronic calculators were simple "four function" devices that did only basic arithmetic such as the Texas Instruments "DataMath" costing $120 in 1972. They were soon followed by the so called "scientific calculator" or "electronic slide rule" that did sophisticated transcendental computations with 8 to 12 digit accuracy. The first scientific calculator was the HP-35 introduced in 1972 (it cost $395). The first desktop computers were slow and had little memory 32K(!), but were powerful and hinted at things to come. In 1979 the first spreadsheet **VisiCalc** was introduced for the Apple II PC and suddenly the world saw a reason to buy a desktop computer!

Scientific calculators are now very inexpensive ($10 to $20) and have significantly changed the mathematics curriculum taught in most countries. For example, we no longer spend valuable lecture time teaching paper and pencil methods evaluate transcendental functions. More time is spent on applications and conceptual understanding of transcendental functions as scientific calculator use has become widespread. Desktop computers have remained expensive and thus still are not used nearly as widely as they should be in the teaching and learning of mathematics in colleges and universities.

Ten years ago calculators took a giant evolutionary step forward and added new software functionality in ROM found only on desktop PC computers. These were the so-called *graphing calculators*, first invented by Casio in 1985. Graphing calculators started a revolution in the teaching and learning of mathematics in the United States and in many other countries as well. Inexpensive graphing calculators were really hand-held computers with built-in graphing software. Graphing calculators could be viewed as *computers available to all students* because of their low cost, ease of use, and portability (Demana, & Waits, 1992).

Based on significant input from teachers using Casio, Hewlett-Packard, and Sharp graphing calculators in 1987-89, Texas Instruments developed its first graphing calculator, the TI-81. The computer graphing software program called Master Grapher that we designed also heavily influenced the features of the TI-81 (Waits & Demana, 1987-1990).

Teachers were immediately impressed with the TI-81, because they easily recognized that its features were designed by teachers for teachers to teach mathematics. It was, first and foremost, a pedagogical tool for mathematics teachers. Texas Instruments quickly followed with the TI-85 by adding an excellent piece of engineering and advanced mathematics software to the features of the TI-81. The TI-85 was regarded as a product for college students because it could solve differential equations graphically and had linear algebra capabilities, but was also used by many AP calculus teachers and students.

Texas Instruments closely followed the progress of teachers and students using the TI-81 and listened carefully to their comments, concerns, and requests. This led to the TI-82, a significant improvement of the TI-81 based on teacher input. The table feature of the TI-82 was a major breakthrough in spreadsheet-like software on an easy-to-use, low-cost hand held product. Next came the TI-80 responding to requests for an affordable graphing calculator for middle school and two-year college students.

Responding to requests from mathematics and science teachers, TI then released the Calculator Based Laboratory (CBL) devices that collected data using the TI-82 or 85. This greatly influenced the use of hand held technology by science teachers. The CBL and the statistics features of graphing calculators have paved the way for regular use of data in mathematics and science classrooms.

The next two products developed by TI were the TI-92 and the TI-83. The TI-92 is based on the Motorola 68000 chip (the same chip as in the Apple Macintosh), and has the features of a state-of-the-art graphing calculator and has built-in powerful computer software for symbolic algebra (from the same authors as *Derive*) and dynamic geometry (*Cabri II*). The TI-83 was designed in cooperation with leaders in the AP Statistics program to include many new important statistics features. It is also an update of the TI-82. Texas Instruments has recently released the TI-86, an update of the TI-85, and the CBR (Calculator Based Ranger), a stand-alone motion detector that connects *directly* to the TI-82, 83, 85, 86, and TI-92.

Impact on Teaching and Learning

Initially, we, as well as many others involved in the modern reform effort, underestimated the impact on teachers of using technology to enhance the teaching and learning of mathematics. We naively referred to this change as an incremental approach to reform. However, the use of technology requires teachers to completely change the way they teach. Teachers also need to be prepared for the complete change in the way students learn using a technological approach. We discovered that most teachers found this change to be revolutionary and not the least bit evolutionary or incremental.

Successful technology based reform requires two important ingredients. First, you need to provide teachers with technology based materials (Demana, Waits, Clemens, & Foley, 1997; Finney, Thomas, Demana, & Waits, 1994). Second, teachers need to be trained in the appropriate use of technology to enhance the teaching and learning of mathematics. We found that we needed to develop a massive professional development program for teachers, that is now called the T^3 (Teachers Teaching with Technology) program (Demana & Waits, 1997).

We observed that surprising few of our students were able to get started on a problem using symbolic techniques. However, virtually all of our students were able to begin the analysis of a problem using numerical or graphical techniques. The numerical or graphical investigation laid the foundation for students to represent and then solve problems using algebraic or analytic techniques. Students, even our remedial students at OSU, became more flexible problem solvers. They were able to attempt problems using a variety of techniques, and did not give up when they were not able to solve problems initially using algebraic or analytic techniques.

Prior to the wide spread use of graphing calculators, we observed that students had virtually no understanding about the use of graphing as a tool to do mathematics. For example, even our best calculus students at OSU did not initially know what it meant to solve an equation of the form $f(x) = 0$ graphically. Their understanding about graphs was so minimal that creating a graph by hand to solve the equation was an impossibly time consuming task. Now, solving equations and other problems graphically, even in algebra courses, is fairly routine. We regularly ask students to solve problems using technology, and to *confirm* the answer algebraically or analytically using traditional paper-and-pencil techniques. Not only do students develop powerful graphical techniques, but teachers can focus on the mathematical reasons that the graphical techniques work.

We also ask students to solve problems using traditional paper-and-pencil methods and then *support* their results graphically. For example, we ask students to use analytic techniques to find solutions to optimization problems or to find coordinates of points of inflection. Then we ask them to support their results by graphing f, f', and f'' in the same viewing window. This helps students to establish the connections among these graphs in an incredibly powerful visual way.

One of the more compelling stories about our earlier work in the C^2PC project was with graphs of rational functions. Our project students, both high school and college, give the following naive description of the class of rational functions. They used paper-and-pencil division to express the rational function *f*/*g* in the form

$$\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)},$$

where *f*, *g*, *q*, and *r* are polynomials with either $r(x) = 0$, or the degree of *r* is less than the degree of *g*. Then, they described the construction of the graph of *f*/*g* as follows: First draw the graph of the polynomial *q*, and then erase a small portion of the graph near each zero of *g*. Complete the graph using the behavior of the function *f*/*g* near the zeros of *g*. In their language, the graph of a rational function is basically the graph of a polynomial except for a few bad places.

Backlash

Most of us can cite our own local examples of backlash against the reform movement. On the national level, there have been numerous articles in the “Notices of the American Mathematics Society” during 1996 and 1997 giving pro and con views about the reform effort and the use of technology. Robin Wilson (February, 1997) gives a glimpse of the division among mathematicians on “reform calculus” in his article in the Chronicle of Higher Education. Speaking against reform calculus:

> “This approach really shies away from anything but superficial use of skills,” says Ralph L. Cohen, a professor of mathematics at Stanford University, which after seven years has decided to stop teaching the ‘reform calculus’ and to move back to something more traditional. “For students who really need to know math and use it, this wasn’t nearly sophisticated or rigorous enough.”

Describing the issues involved in the debate:

> “The debates are as deep as those between two different religious groups,” says Ronald G. Douglas, provost at Texas A&M University, who is considered the father of the reform movement.

Speaking in favor of the reform movement:

> “They (students using traditional methods) learned they could stick in a couple of key symbols, statements, and equations and put forward what were found to be acceptable solutions, even though they had no idea what was going on,” says Morton Brown, a professor of mathematics at the University of Michigan.

It is our opinion that much of the debate revolves around misconceptions on both sides about the goals of the reform effort. It is certainly not the goal of the reform effort to abandon algebraic or analytic techniques, at least not in our opinion. Yet teachers sometimes give this impression or mistakenly believe that this is true in their zealous advocacy for the use of technology. For example, the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) clearly states that certain algebraic techniques should receive decreased attention in the curriculum. Some teachers wrongly infer that if less attention is good, then no attention is better. This is one fundamental misconception about the NCTM Standards. A careful reading shows that this was never the intention of the authors of the Standards. We say more about this issue in the next section on a balanced approach to curriculum reform.

A Balanced Approach to Curriculum Reform

Prior to the advent of easy to use hand held technology, about 85% of the mathematics curriculum consisted of paper-and-pencil computation. The computation involved the algebraic and analytic process of mathematics including the common symbolic manipulations of algebra and calculus (by paper and pencil). This computation usually involved very low order thinking skills, and often have been associated with the phrase “drill and kill mindless manipulations.” In the pre hand held technology curriculum, there were precious few application examples and they almost always occurred as consequences of mathematics concepts developed algebraically or analytically. Further, little or no real proof occurred in the standard courses. There is growing evidence that paper and pencil manipulation skill alone does not lead to better understanding of mathematical concepts. Indeed, the use of hand held technology can provide more classroom time for the development of better understanding of mathematical concepts by eliminating the time spent on “mindless paper and pencil manipulations.”

The advent of affordable scientific calculators and graphing calculators allowed students to make widespread use of numerical and graphical techniques. Suddenly, students were able to make regular use of numerical and graphical problem solving techniques. There is widespread agreement that technology, problem solving, and the use of real data has an expanded role in the current mathematics curriculum. We expect these three areas will play a more important role, and paper-and-pencil computation will occupy a smaller share of a balanced modern curriculum.

Many mathematicians are becoming increasingly concerned with the perceived lack of attention to paper-and-paper skills in the new evolving mathematics reform curricula, as we stated earlier. Nothing could be further from the truth. Paper-and-pencil skills are and will continue to be an important part of the curriculum. However, the role of paper and pencil computation will change dramatically in the future because of hand-held technology. Technology provides a “better mouse trap” for much computation. We must recognize and exploit this fact.

Our new challenge is to think about computation differently. Each paper-and-pencil algorithm should be analyzed to see if the procedure contributes any understanding to the process. If not, it should be removed and performed with technology. For example, there is probably widespread agreement that the square root algorithm and finding trigonometric and logarithmic values from a table by interpolation are obsolete. The concept of interpolation is not obsolete as it is an important idea in mathematics. Using interpolation to find values of trigonometric and logarithmic functions from a table is obsolete. Hand held computer symbolic algebra (CSA) will soon make many of the paper-and-pencil factoring algorithms obsolete but not the process of factoring (which is a key concept in the fundamental theorem of algebra). The same will be true for many of the paper-and-pencil symbolic procedures typically taught today.

We believe computation should be done in one of the following three ways today and in the future. (By computation we mean those manipulative procedures associated with paper-and-pencil arithmetic, algebra, and calculus.)

1. Mental computation

2. Paper-and-pencil computation
3. Computation done with technology

Our challenge is to decide when a given computation method is appropriate. We believe it will often turn out that certain computations are judged to be mental or paper-and-pencil computation in one course (or section of a course), but then should be done with technology in subsequent courses (or section of the course). This pedagogical technique is called the *white box/black box* principle. For example, partial fraction decomposition in calculus is a “black box” procedure best done with technology. But integration of functions is a white box procedure (using paper and pencil). That is, we allow the use of some algebraic, non-calculus, black box procedures while not allowing any black box integration procedures (until the skill or concept is learned). The white-box/black box principal was first introduced by Professor Bruno Buchberger from the Research Institute for Symbolic Computation in Linz, Austria. This wonderful principle is outlined in detail along with other excellent examples in the book by Heugl, Klinger, and Lechner (Addison Wesley, 1996).

We also need to analyze paper-and-pencil procedures to see if technology can add understanding about the underlying concepts. The following example illustrates that technology seems to deepen student understanding about one paper-and-pencil process.

Example. Solve the system of equations:

$$3x + 4y = 5$$

$$2x - 3y = 4$$

Solution: We use substitution, an important mathematical process, to solve for *x* and *y*. The first line of Figure 1 uses the TI-92 to solve the first equation for *y* in terms of *x*. The second line defines *y* as a function of *x*. This is a step that is not very well understood by many students when they use paper-and-pencil techniques to solve the system. Understanding this TI-92 step contributes to student understanding about the process.

In the third line, we solve the second equation for *x*. Because we have defined *y* to be a function of *x* we should expect to get a number. Again deeper understanding is possible for students. The fourth and fifth lines obtain the values of *x* and *y* that solve the system.

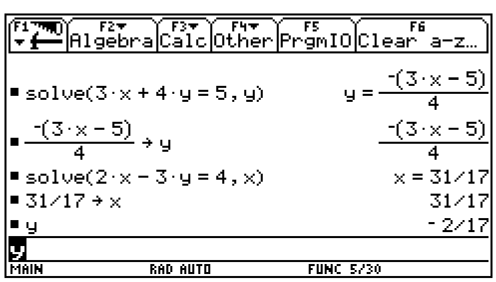

Figure 1. Solving a system of equation using the TI-92.

All students can then use this procedure to solve a general system of two linear equations in two variables and discover Cramer's Rule. This type of computation does not happen in a non technology paper-and-pencil enviroment.

Proof, or giving convincing arguments, is another area of the curriculum that can be expanded through the use of technology. There is precious little of this in a standard mathematics curriculum except possibly in geometry courses. Even in most geometry courses, students are really only committing facts to memory that are later spewed out on examinations.

The ability to obtain many correct symbolic results is an important feature of technology. Students can obtain many correct statements quickly to help them form correct generalizations. This type of algebraic exploration is not possible for many students using only paper-and-pencil techniques. However, with technology, all students can experience symbolic exploration. For example, Figure 2 shows three cases of the following identity using the TI-92 computer algebra features.

$$\sin(nx) + \cos(nx) = \sqrt{2}\,\sin(nx + p\ / 4)$$

Now, with teacher encouragement and guidance, students could use the sine of the sum of two angles formula to analytically confirm or prove the above identity.

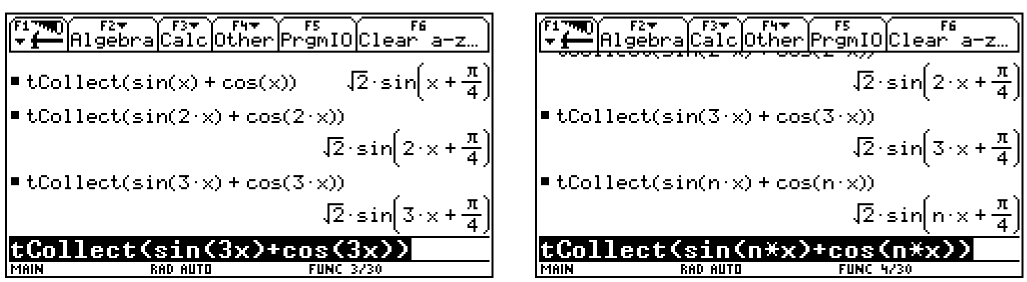

Figure 2. Using the trigonometric computer algebra tCollect(command of the TI-92.

With appropriate use of technology, all students can participant in a balanced mathematical curriculum that has problem solving, computation, and proof as major themes. Appropriate use of scientific calculators and graphing calculators helped empower all students numerically and graphically. Now appropriate use of CAS can empower all students symbolically.

The Future

Prior to graphing calculators, teachers and professors had to rely exclusively on expensive computers (usually housed in a separate computer laboratory) to deliver computer enhanced visualization in mathematics teaching and learning. Only a few elite schools, colleges, and universities could provide such an experience to all mathematics students on a regular basis. A CAS, available usually only on expensive PC's, generally consists of three main software packages – symbol manipulating software, numerical solvers, and computer graphers. Graphing calculators provided two of these (all but the symbol manipulating software) at far less cost and often in a more user friendly environment. The pedagogical significance to the mathematics community of the small, inexpensive, hand held graphing calculators should not be under estimated. It is now widely accepted that graphing calculators provide millions of students useful and exciting experiences enhancing their mathematics learning with computer visualizations. Teachers are now able to present mathematical ideas, concepts, and applications in both traditional symbolic as well as computer generated numerical and graphical representations. Powerful new approaches to learning mathematics have been made possible by graphing calculators. It is now well established in many countries that a richer mathematics curriculum is possible when all students have access to graphing calculators.

In his opening Keynote Address at a conference in Germany one of the authors recently attended, Professor Benno Fuchssteiner (author of a popular new CAS software called *MuPad*) put forward the case that, ideally, students should learn mathematics using the same tools as they will go on to use in their future applications of mathematics. While this is clearly a very laudable aim, its implementation looks likely to be restricted to only the richer institutions, at least for the medium term. Our experience is that there are many teachers who would like to be able to use CAS, but for whom the ideal is unachievable. This is why hand-held technology like graphing calculators had a major part to play in the past 10 years and why, in the future, hand-held CAS tools will be so very important.

Just as CAS has come a long way, so has pocket-sized hand held technology with battery power, LCD screen and robust casing. Maybe once it was possible to distinguish between an electronic calculator and a personal computer – but the technologies have now converged to the extent where such distinctions are almost irrelevant. Yet there seems to remain a dichotomy in the minds of some of those who make vocal interjections about the state of mathematics teaching today between, on the one hand, the calculator – which is perceived as the source of all evil – and, on the other, the computer – which is revolutionizing the way in which mathematics is going forward. So, just to confuse the situation, we now have tools, like the Texas Instruments' TI-92, which have features of both!

Graphing calculators, when compared to hand-held CAS like the HP-48, TI-92, and the new Casio CAS calculator, do not lead to significant change in core curriculum content. Graphing calculators are wonderful tools that can be used to *enhance, not replace*, the current core curriculum. They enable better pedagogy for teaching mathematics, facilitate the incorporation of problem solving activities and applications, provide a motive for asking students to think about mathematical concepts and use them to justify the use of technology, and help students learn to value mathematics. But graphing calculators do not, nor will not, cause significant core curriculum change. The graphing calculator revolution was teacher driven (teachers required their use and purchase) rather than student driven (students purchase them because they see their

Hand-held CSA tools will no doubt become the scientific calculator of the future, and

CSA tools *will dramatically change*
curriculum taught in grades 9-16 in the next 5-25 years in this country and around the world. We do not know yet exactly how we will get there. A great deal of experience and

testing in the presence of graphing calculators will be mild compared to testing in the presence of CAS. We need to begin today to plan for this next dramatic step in the

Research Results

We, and the many teachers working with us in our projects, have observed students

found in newspapers and other sources, as well as data collected using CBLs give students a better feeling about mathematics. We now see how a technology enhanced approach to

Many mathematics education research studies have been conducted as an outgrowth of the C PC project. There is a summary of several of these studies in a paper by Dunham

Technology in Collegiate Mathematics" that was founded by Waits and Demana. In fact, there are numerous research reports that appear in the proceedings of this annual conference, the tenth of which was held in 1997.

An excellent source of information about issues related to use of technology in instruction is contained in the book titled, "Impact of Calculators on Mathematics

in the field:

1. The impact of theory and research from
   use of calculators in mathematics classrooms.
2. Research studies needed in the area of staff development and training.

4. Need for more programmatic research in this area.

We recommend that this book be read from cover to cover.

changed his attitude about algebraic thinking. It shows how difficult it is for teachers to change the way they teach. This study certainly supports the need for research studies in the area of staff development and training called for by Bright, Waxman, and Williams (1994). Interesting, the teacher in this study is now one of our national T^3 Instructors.

Another important source of information about use of technology is the book titled, “Integrating Research on the Graphical Representation of Functions” (Romberg, Fennema, & Carpenter, 1993). This book is concerned with the integration of research on teaching, learning, curriculum, and assessment with respect to the graphical representation of functions. In his chapter, Jim Kaput argues that research in the representation of quantitative relationships should be further ahead of current practice than it is now in at least three dimensions: technology, curriculum, and representation. This is another book that warrants reading from cover to cover.

The process of change called for by the participants at this conference goes beyond what researchers and curriculum developers can do by themselves. This is an issue for parents and all of society. Peressini (1997) describes the importance of parental involvement in the reform of mathematics education.

Our community needs to collect and have at easy access many studies that show the value of teaching and learning with technology. We also need to show that appropriate use of technology does not decrease paper-and-pencil skill. This is the only way we will be able to combat the backlash effort in a positive way. Concentrating on collecting this evidence is our next important challenge.

Summary

One of the editors for the article by Demana et al (1990) listing the 16 recommendations for a perfect calculator was M. Kathleen Heid. As an editorial reflection of our article, she stated, in part:

> “As more and more students and teachers use these calculators in mathematics learning, the need for a changed, technology-responsive curriculum will become apparent. Nor only will new topics and approaches be needed, but the inclusion in school curricula of some of the old topics and approaches will be viewed with increased skepticism. As calculators couple graphing capabilities with improved symbolic manipulation, entire courses and sequences of courses will need to be rethought.”

We certainly agree with this observation even more so today.

We believe what is needed today is a school and university mathematics curriculum that takes advantage of computer technology to assist students in gaining mathematical understanding, in becoming powerful and thoughtful “thinkers,” communicators, and problem solvers. We seek a *balanced approach* to the use of technology.

We have always tried to emphasize balance. Our well known graphing calculator C^2PC philosophy was a call for balance.

DO algebraically with paper and pencil, and then the result with computer graphing,
using computer graphing, and then **CONFIRM**
pencil algebra,
DO

In the past there was no real balance. In the future there will still be a need for mental mathematics skills (perhaps even a greater need than in the past), some paper-and-pencil

calculus manipulative skills.

Our community can how student use of computer symbolic algebra and computer interactive geometry impact the mathematics curriculum. This new generation of hand-held student computer tools will no doubt become as popular as
must deal with the fact that computer symbolic algebra
– far better
doing many of the "manipulations" associated with mathematics.

These new tools can also be used to illustrate important concepts and applications of mathematics. We must redefine "basic skills" to include those paper-and-

Of course, some traditional paper and pencil skill will continue to be necessary for mathematical activities as will traditional mental skills. However, we *must* agree to stop spending large portions of our time teaching obsolete paper-and-pencil algebra and calculus manipulations. These obsolete paper-and-pencil skills be identified we must determine the essential paper-and-pencil computation skills.

References

Bright, G. W., Waxman, H. C., & Williams, S. E. (Eds) (1994),
Mathematics Instruction. Lanham, Maryland, University Press of America.

developer. *Arithmetic Teacher, 34*

Demana, F., & Leitzel, J.R. (1988). Establishing fundamental concepts through numerical problem solving. In A. F. Coxford & A. P. Shulte (Eds.), *K-12: 1988 yearbook* (pp.61-88). Reston VA: National Council of Teachers of

Demana, F., Dick, T., Harvey, J., Kenelly, J., Musser, G., & Waits, B. K.. (1990). Graphing Calculators: Comparisons and Recommendations. *The Computing Teacher,* Vol 17 (7), 24-31.

Demana, F., & Waits, B. K. (1992). A Computer for all Students. *Mathematics Teacher,* 82(1), 94-95.

Demana, F., Waits, B. K., Clemens, S. R., & Foley, G. D. (1997). *Precalculus, A Graphing Approach*, 4th ed. Menlo Park, CA. Addison Wesley.

Demana, F., & Waits, B. K. (1997). A Zero-Based Technology Enhanced Mathematics Curriculum for Secondary Mathematics. In A. Ralston & H. Burkhardt (Eds.), Proceedings of WG 11, ICME 8. England.

Dunham, P. (1992). Teaching with Graphing Calculators: A Survey of Research on Graphing Technology. In L. Lum (Ed.), *Proceedings of the Fourth Conference on Technology in Collegiate Mathematics,* 89-101. Reading, MA, Addison Wesley.

Finney, R. L., Thomas, Jr., G., B. (1994). *Calculus, Graphical, Numerical, Algebraic*. Reading, MA., Addison Wesley.

Heugl, H., Klinger, W., & Lechner, J. (1996). *Mathematikunterricht mit Computeralgebra-Sytemen: Ein Didaktisches Lehrerbuch mit Erfahrungen aus dem osterreichischen DERIVE-Projekt*. Bonn, Germany, Addison Wesley

Mathematical Association of America. (1986). *Toward a Lean and Lively Calculus, Report of the Conference/Workshop to Develop Curriculum and Teaching Methods for Calculus at the College Level*, MAA Notes No 6, R. G. Douglas (Ed). Washington, D.C., MAA.

Mathematical Association of America. (1987). *Calculus for a New Century, A Pump, Not a Filter*, MAA Notes No 8, L. A. Steen (Ed). Washington, D.C., MAA.

National Council of Teachers of Mathematics. (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA, NCTM.

Nicol, M. P. How one Physics Teacher Changed His Algebraic Thinking. *Mathematics Teacher,* 90(2), 86-89.

Peressini, D. Parental Involvement in the Reform of Mathematics Education. *Mathematics Teacher*, 90(6), 421-427.

Romberg, T. A., Fennema, E., & Carpenter, T. P. (Eds) (1993). *Integrating Research on the Graphical Representation of Functions*. Hillsdale, New Jersey, Lawrence Erlbaum Associates.

classroom. *American Mathematical Monthly*, Washington, D.C., MAA.

Apple II. Reading, MA. Addison Wesley.

(C[2]
Williams (Eds), *Impact of Calculators on Mathematics Instruction*
University Press of America.

Charge – Mathematicians divide over a curricular movement that some say has cheated students. *Chronicle of Higher Education*, February 7, 1997.