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Enhancing Mathematical Concepts through Leading Questions and Hand-held Data Collection Tools

Edward D. Laughbaum
The Ohio State University
231 West 18th Avenue
Columbus, Ohio 43210 USA
www.math.ohio-state.edu/~elaughba/

Abstract:
Hand-held data collection technology allows for access to real-world data collection – at any other time and almost any place. Is the use of data, and its collection, desirable to the mathematical leaning process? The answer is a resounding yes! Not only can we teach significant mathematical ideas in the process; we also help our colleagues in the sciences. When done correctly, students actively involved in the data collection process take ownership of the data and of the mathematical learning that follows. Ownership, in turn, leads to understanding – a key component that may be missing in traditional instruction.

Students and teachers must know how to use the software and follow proper data collection techniques. Both of these can be learned with practice in doing data collection. There are marvelous opportunities for you to collaborate with a science teacher. Data collection is a learned skill, and many times mathematics teachers don't have the "lab" skills that science teachers have because, traditionally, mathematics has not been thought of as an experimental science. Teachers must also be able to ask good questions to help guide and direct students as they start an activity. If used as a demonstration in class, the teacher must be able to lead students to the desired mathematical outcome - understanding a mathematical concept.

Mathematical Topic: *Asymptotic behavior- vertical asymptote*

Equipment needed: CBL 2™, TI calculator, DataMate software app, Vernier pressure sensor.

Information: The CBL 2™ will collect volume (in cc) and pressure (in ATM or other units) data for the air secured in a 20-cc syringe.

Setup: Fill syringe with 20-cc of air. Wait a second for the pressure to stabilize and close the valve. Attach the sensor cable to CH 1 on the CBL 2 and run the DataMate app. Since the Pressure Sensor is not auto-ID, select 1 for Setup. With the cursor on CH 1, type ENTER and select Pressure, followed by Pressure Sensor. Select the units of measure (ATM or your choice). Move the cursor to MODE followed by the ENTER key. Select EVENTS WITH ENTRY. Type 1 for OK. At this time, DataMate will display the pressure in the top right corner of the calculator screen. When ready to record data points select 2 (Startup) and follow screen directions. (Note: Enter Value means the volume of the air at the current pressure.)

Mathematics Pre-requisites: Students should have some experience in developing linear and quadratic models from a data set using their understanding of the connections between function parameters and the resulting function behaviors. There is no need for students to use regression models. This activity is intended to introduce students to their first function that displays vertical asymptotic behavior.

Leading Questions:

- As you cause the volume to approach zero with your hand, what do your senses tell you about the pressure?
- Can **you** change the volume to 0 cc in the syringe? (without damaging the equipment)
- If you could change the volume to 0 cc, what would happen to the pressure?
- Do you think the relationship between volume and pressure linear? That is, for any ΔV and related ΔP is the ratio $\frac{\Delta P}{\Delta V}$ the same?
- Might the relationship between volume and pressure be quadratic? Explain.
- Describe a trend (pattern) you see in the **change in the pressure** on the sensor as you **change** the volume from 20 to 19 cc, from 19 to 18 cc, from 15 to 14 cc, from 10 to 9 cc, from 6 to 5 cc. (The pressure is continuously displayed by the DataMate software.)
- As the volume approaches zero, describe the general behavior of the rate of change of the pressure.
- Finally, it is the time to collect and store the data points. Perhaps at 20, 18, 15, 13, 10, 8, and 5 cc's. Be sure to let the pressure sensor stabilize at each volume before collecting the data.
- What might the pressure be at 3 cc of volume? 2 cc of volume? (This assumes that most people cannot get the volume to 2 or 3 cc. It may also provide motivation for the need of the symbolic form of the relationship.)
- Graph the data. Does the graph behave as you expected?
- Summarize the behavior of the graph of the relationship as the volume approaches zero.
- Do you notice any pattern developing between the x and y-coordinates?
- Multiply the x and y-coordinates together and store the results in L_3.
- Do you notice any pattern developing between the x and y-coordinates now?
- Make a suggestion for a symbolic model of the data relationship and graph it.
- How does the graph of the model relate to the data points?

You are now ready to analyze the behavior, and functions with this behavior, using a more formal mathematical approach.

Mathematical Topic: *Piece-wise defined functions*

Equipment needed: CBL 2™, temperature probe, TI calculator, a hot hand

Information: The CBL 2 is going to collect time-temperature (either °C or °F) data as the temperature probe is heated in my hand for 30 seconds and then cooled in the air for 30 seconds.

Setup: The stainless steel temperature that comes with the CBL 2 is auto-ID. When it is connected to CH 1 (or other) it will be identified by DataMate. However you must still run Setup to change the mode to Time Graph. Type 1 for Setup and then move the cursor to Mode. Type ENTER and select Time Graph. Type 2 for Change Settings and follow the on-screen directions. A suggested time between samples is 1 second and collect 60 samples. If you want to switch units, move the cursor to CH 1 (type ENTER) and select the temperature probe followed by Stainless Temp (F or C). When finished setup, type 1 for OK, and 1 for OK to continue. From the main screen, type 2 to start collecting sample points. Put sensor in your hand for 30 seconds and then wave in the air for 30 seconds.

Mathematics Pre-requisites: Students should have some experience in developing models from a data set using their understanding of the connections between function parameters and the resulting function behaviors. There is no need for students to use regression models. This activity is intended to introduce students to the need for a function that does not behave as do simple elementary functions typically studied in pre-calculus courses.

Leading Questions:

- Do you think the heating portion of the time-temperature graph will display similar behaviors as the cooling portion?
- Will the graph be symmetric to a vertical line through 30 seconds? Why?

Collect data now.

- After seeing the visualization of the data set, what function have you studied that behaves in this fashion?
- Do you think it is possible to have a single function that could be used to model this data set?
- What kind of function is suggested by the data for the heating portion of the data set? Find it. (Or run the program called HOTHAND.)
- What kind of function is suggested by the data for the cooling portion of the data set? Find it. (Or run the program called HOTHAND.)
- If you add the two functions above, will the graph of the sum be a good model of the data?
- Is it important to control the domain of either model? How can you control the domain of a function?
- What is the domain of the function $y = 2x - 3 + 0\sqrt{x-1}$?
- Is the graph of $y = 2x - 3 + 0\sqrt{x-1}$ linear?
- Propose a solution to the problem of not being able to model the data with a single elementary function.

Mathematical Topic: *Maximum, increasing, decreasing, zeros*

Equipment needed: CBR, (preferably used with the CBL 2) TI calculator, light-weight ball (soft)

Information: The CBR will collect time-distance data, as a ball is tossed straight up above the CBR lying directly below on the floor. Time will be in seconds and distance in meters.

Setup: If you are using the CBR with the CBL 2, attach the cable to the sonic port of the CBL 2 and the CBR. (Special cable not needed with a motion sensor.) The CBR is auto-ID, so after starting DataMate it will know it is attached. However, you must still run Setup to change to settings on Mode. Set the Time Graph to 0.05 second between samples with 50 samples. After setting mode type 1 for OK for Time Graph and 1 for OK to set Mode. On the main menu, type 2 to start collecting data. You will hear two beeps (about a second or so after typing the Start command) indicating data collecting is beginning. Use Select Region to isolate the data points that show the actual flight of the ball. To see a graph, type the ENTER key after moving the cursor to the desired type. Trace may be used. To exit a graph, type the ENTER key.

Mathematics Pre-requisites: Students should have some experience in developing linear and quadratic models from a data set using their understanding of the connections between function parameters and the resulting function behaviors. There is no need for students to use regression models. This activity is intended to promote understanding of the topics. However, this activity (excluding the modeling part) might be used with no pre-requisites other than a basic recognition that some data sets when graphed display a variety of common behaviors. In this case, it can be used as the first contact students have with the concepts of increasing, decreasing, maximum, and zeros. Actually, this activity contains too many objectives for a good lesson plan and is used here as a demo only.

Leading Questions:

- As I toss the ball straight up, describe the time-height relationship (as best you can) in mathematical terminology. (Do not use the words “up” or “down.”)
- Does the ball reach a point where it doesn’t go any higher?
- As time changes from 0 second to when the ball is at the high point, how is the height of the ball changing?
- As time changes from when the ball is at the high point to when it is on the ground, how is the height of the ball changing?
- How might you describe the height of the ball when it is between “increasing” in height and “decreasing” in height?
- What is the height of the ball when it is on the floor (relative to the CBR)?
- If you want to find a function that models the data from above, how would you select a suitable candidate?
- Find a model of the data collected above. (Or run BALLTOSS Program.)
- When is the model “increasing?”
- When is the model “decreasing?”
- When does the model have a maximum value?
- When does the model have a value of zero?
- Given the function $Y_2(x) = -4.7(x - .16)^2 + 5.1(x - .16) + 0.4$, describe when it is increasing, decreasing, what the maximum value is, and find the zeros. (Ask this only if BALLTOSS is used.)

Mathematical Topic:	*<u>Constant rate of change: definition of linearity</u>*
Equipment needed:	CBR, TI calculator, CBL 2 (optional)
Information:	The CBR will collect time (in seconds) and distance from the CBR (in meters) data for an object in the path of the CBR. Collection will start at the push of the ENTER key and run for a specified length of time. The "object" will be the presenter walking at a constant rate.
Setup:	From the DataMate app, select Setup to set Mode to Time Graph with 0.5 second between sample points for 8 samples. From the main menu, type 2 to collect data. The CBR will beep about 1 – 2 seconds after you type 2 indicating the beginning of data collection. After viewing the time-distance graph, if you want to re-do the data collection go to the main menu and type 2 again. When you exit DataMate, please note where data is stored. After exiting, you are ready for the questions below. Another option for data is to run the program CONSTRAT. (Time in L_1, distance in L_2, velocity (rate) in L_3.)

Mathematics
Pre-requisites: This activity is intended to promote understanding of a constant rate-of-change and its connection to the linear function. This activity might be used with no pre-requisites other than having an idea on how to calculate and average rate of change.

Leading Questions:

- Any ideas on how we can find my average speed? If so, how? (Collect data first.)
- Here is another idea, find the change in position divided by change in time for each data pair. Find the average of these. ($\Delta L_2/\Delta L_1$)
- If the speedometer on your car reads 80 km/hr as you drive, are you moving at a constant rate?
- Was I walking at a constant rate?
- Consider the graph of the data and look at the meaning of ΔP and Δt. (ΔL_2 and ΔL_1) (use DRAW)
- What would the graph look like if I walked at a faster rate of change (speed)? That is, what if ΔP is greater for the same Δt?
- What would the graph look like if I varied my speed (rate of change)?
- Based on this discussion, propose a method you could use to decide if a relationship is linear.
- Define (intuitively) rate of change.
- Would the distance be increasing if I walked toward the CBR?
- How can I create an increasing graph (function), a decreasing graph (function), a constant graph?