Curriculum and assessment congruence - Computer Algebra Systems (CAS) in Victoria

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Abstract

Students undertaking mathematics courses in their final year of secondary education in Victoria are assessed using a combination of school based coursework assessment and examinations. Over the last decade, students have used technology such as graphics calculators, statistical software, spreadsheets and computer algebra systems extensively in tackling extended investigative, problem solving and modeling tasks as part of their school based assessment. The use of graphics calculators has been permitted in calculator *neutral* mathematics examinations since 1997, while from 2000, these examinations will be set with *assumed* student *access* to an approved graphics calculator and will also include some calculator *active* questions. This development has been supported by the implementation in 2000 of a revised senior mathematics study that includes a technology active learning outcome for all mathematics courses. The University of Melbourne, in partnership with Board of Studies, Victoria, and three calculator companies has received a major Commonwealth government three year research grant 2000 – 2 to undertake a pilot study (http://www.edfac.unimelb.edu.au/DSME/CAS-CAT) for a CAS active mathematics course and corresponding CAS active examinations, and to plan for its possible accreditation within a revised mathematics study. Congruence between curriculum, pedagogy, assessment and values is a key issue for the related discourse. In this presentation some important connections between these elements and associated issues for systems and policy makers will be discussed.

Introduction

The following discussion draws together key ideas and material from two papers, the first is a keynote address by the author '*Technology in assessment and learning in senior secondary mathematics – some reflections*' for the Australian Association of Mathematics Teachers (AAMT) Virtual Conference 2000, http://www.aamt.edu.au/VC2000. This conference will run from July 28 until September 4. The second paper '*Research-led policy change for technologically-active senior mathematics assessment*'(Stacey, McCrae, Chick, Asp and Leigh-Lancaster) was presented at the Mathematics Education Research Group of Australasia (MERGA) Conference in July 2000 and will be published in the proceedings of the conference.

The use of technology is an integral component of the revised Victorian Certificate of Education (VCE) Mathematics study 2000 – 3. While the courses of the revised study are mainly refinements of the previous courses and retain their analytical components, some graphics calculator 'active' content has been introduced into the areas of study and outcomes for these courses. Mathematics examination setting panels for Unit 3 and 4 courses (typically undertaken by Year 12 students) assume that all students will have access to an approved graphics calculator. This follows on from an interim period of several years where the use of graphics calculators has been permitted in mathematics examinations. During this time teachers have had access to extensive professional development in the use of graphics calculators, in particular through the Board of Studies working in partnership with the Mathematical Association of Victoria (MAV) and university education faculties, as well as receiving ongoing advice from the Board of Studies. Publishers and their authors have explicitly included technology related material, especially for graphics calculators, in their text series for the revised courses and in many cases these are supported by CD ROM based resources. Calculator companies have also been active in developing teacher resources and providing professional development.

As is the case in various systems, there are two components to the formal assessment of student work – examinations (66% for VCE Unit 3 and 4 mathematics) and school based coursework assessment (34% for VCE Unit 3 and 4 mathematics). There are no restrictions on the use of graphics calculator or other technology such as statistical software, dynamic geometry software, computer algebra systems or spreadsheets in coursework assessment. In the revised VCE mathematics study, coursework assessment consists of a limited number of tasks of specified types, assessed by teachers in accordance with weights for a set of three outcomes and mark allocations for criteria related to these outcomes.
Teachers devise suitable tasks and assess student work, using various resources, including the Board of Studies *Assessment guide* and *Implementation kit* for Mathematics, and other Board advice published in the *VCE Bulletin* and on the Board's website http://www.bos.vic.edu.au .

What is meant by 'the use of technology'?

Teachers require clear advice on the expected use of technology and the revised VCE mathematics study includes a technology specific outcome for all mathematics units. For Mathematical Methods Units 3 and 4, the mainstream VCE Mathematics functions, coordinate geometry, algebra, calculus and probability course this outcome (Outcome 3) is:

On completion of this unit the student should be able to select and appropriately use technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modeling or investigative techniques or approaches.

Each outcome is elaborated by a set of key knowledge and key skill statements, which were devised by accreditation panels for each course, with significant input from practising teacher members of these panels.

Key knowledge

To achieve this outcome the student should demonstrate knowledge of

- *exact and approximate technological specification of mathematical information such as numerical data, graphical forms and the solutions of equations;*
- *domain and range requirements for the technological specification of graphs of functions and relations;*
- *the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions or equations;*
- *the similarities and differences between formal mathematical expressions and their representation in various technology applications;*
- *the appropriate selection of a technology application in a variety of mathematical contexts.*

Key skills

To achieve this outcome the student should demonstrate the ability to

- *distinguish between exact and approximate technological presentations of mathematical results, and interpret these results to a specified degree of accuracy;*
- *produce results using technology which identify examples or counter-examples for propositions;*
- *produce tables of values, families of graphs or collections of other results using technology which support general analysis in problem solving, investigative or modelling contexts;*
- *use appropriate domain and range specifications which illustrate key features of graphs of functions and relations;*
- *identify the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions or equations;*
- *specify the similarities and differences between formal mathematical expressions and their representation in various technology applications;*
- *make appropriate selections for technology applications in a variety of mathematical contexts, and provide a rationale for these selections;*
- *relate the results from a particular application to the nature of a particular mathematical task (investigative, problem solving or modelling).*

To assist teachers in devising suitable tasks for coursework assessment that incorporate the use of technology, the corresponding criteria in the *Assessment guide* relate to the extent to which the student response to a task demonstrates:

Criterion 1

Appropriate selection and effective use of technology.
Relevant and appropriate selection and use of technology, or a function of the selected technology for the mathematical context being considered. Relationship of the results from an application of technology to the nature of a particular mathematical question, problem or task. Use of appropriate specifications which illustrate key features of the mathematics under consideration.

Criterion 2

Application of technology.
Production of tables of values, families of graphs, diagrams or collections of other results using technology which support analysis in problem-solving, investigative or modeling contexts. Production of results efficiently and systematically which identify examples or counter-examples which are clearly relevant to the task.

Congruence between curriculum, pedagogy and assessment

The issue of *congruence* between the use of technology to carry out various activities and the similar use of such technology in the assessment of *performance* related to these activities is a key element in the current discourse in mathematics education on the impact and role of graphics calculators and other technologies. There are several components to the use of technology in senior secondary mathematics - teacher instruction, student learning, student use in school based (coursework) assessment and student use in systemic examinations. Such use could be banned, permitted or expected. Each of the levels of increasing overall use shown in *Figure 1* can be considered from the perspective of congruence between these components. A key principle is the assumption that students have had sufficient opportunity to become conversant with required applications of a given technology before its use in assessment. It is also assumed that teachers would be responsible for providing students with instruction in the use of a given technology application, where this technology is to be subsequently used by students.

Level of overall use	Teacher use in instruction	Student use in learning	Student use in coursework assessment	Student use in examinations
0	✗	✗	✗	✗
1	✓	✗	✗	✗
2	✓	✓	✗	✗
3	✓	✓	✓	✗
4	✓	✓	✓	✓

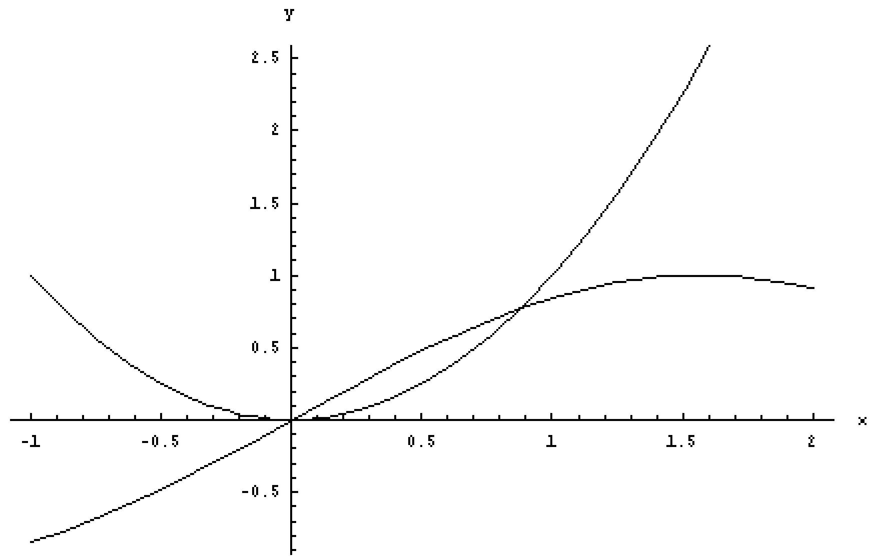
Figure 1 - *components of levels of technology use*

For example, in Victoria in 2000, the level of overall use of graphics calculators is level 4, and some questions that require the use of this technology for numerical equation solving, differentiation and integration could well be asked, as applicable, in mathematics examinations. It is important that teachers are provided with appropriate sample questions, as in the February 2000 *VCE Bulletin Supplement 1* (*Sample examination material*) and related subsequent advice such as the following excerpt from the June 2000 edition of the *VCE Bulletin:*

> Students will need to be able to use *analytical, numerical* or *graphical* approaches to tackle questions, consistent with the areas of study and outcomes for Mathematical Methods in the mathematics study design. Where a numerical answer to a question, or part of a question, is required, this may be obtained using any of these three approaches as appropriate unless instructed otherwise. Students will need sufficient opportunity to become conversant with each of these approaches to identify which approach is likely to be more effective or efficient in a particular context.

There may well be some questions where no readily accessible analytical approach or no analytical approach, is available, in other instances it may be the case that only an analytical approach will be suitable. Where a numerical answer is required, students are expected to provide the answers correct to the specified accuracy. For example, the gradient of the tangent to the curve with rule $y = x^2\ 2^x$ at $x = 1$, *correct to three decimal places*, is 5.386.

If the instruction *use calculus* is given, students are expected to show an appropriate derivative or anti-derivative expression in their working. For example, consider the functions with the rules $y = x^2$ and $y = \sin x$. Part of the graphs of these two functions is shown on the axes below:



The graphs of the two functions intersect only twice, at $x = 0$ and $x = k$, where $0.5 < k < 1$. A numerical or graphical approach is required to find $k = 0.877$, correct to 3 decimal places.

The area between the two curves from $x = 0$ to $x = k$, correct to 3 decimal places, can be found by evaluating the definite integral $\int_0^k (\sin x - x^2)dx$. The instruction *use calculus* to find the area between the two curves requires the anti-derivative of the function in the integral to be shown, such as in the expression:

$$\left[-\cos x - \frac{x^3}{3}\right]_0^k$$

The value of the definite integral, 0.136 correct to 3 decimal places, can be evaluated by substitution of the values for $x = 0$ and $x = k$ into this expression, or by numerical integration using a graphics calculator in the form of a built in routine or a previously entered program.

Where an *exact value answer* is required to a question, or part of a question, a numerical approximation to this value will not be accepted. For example, if the exact value of the derivative of tan $2x$ at $x = \frac{p}{12}$ is asked for, a decimal approximation such as 2.6667, is not an acceptable response, the exact value of $\frac{8}{3}$ or $2\frac{2}{3}$ is required, with appropriate working.

Congruence was an important consideration in the Board's rationale to move to an 'assumed access' status for graphics calculators in VCE mathematics examinations " *…the Board decision also more closely aligns student use of graphics calculators in activities related to learning mathematics with corresponding use of graphics calculators in assessment of this learning*" (page 7, *VCE Bulletin*, October 1998).

Computer algebra

Hand held technology such as 'super-calculators' which are capable of symbolic manipulation involving many routines of algebra and calculus currently learnt at school is already available, and is likely to become sufficiently affordable for most students over the next few years. The use of such technology poses various challenges for mathematics curricula and approaches to teaching, learning and assessment, in particular for examinations. Teachers and students in some Victorian schools have been using computer algebra systems (CAS) since the early 1990's, with some students using them in tackling the extended investigative or problem solving common assessment tasks of the previous mathematics study. While these extended tasks have been replaced by a collection of a limited number of smaller coursework assessment tasks (tests, analysis tasks, application tasks) in the revised mathematics study, students can still used CAS in tackling these tasks.

From 2000 – 2002 the Board of Studies in Victoria, in partnership with Department of Science and Mathematics Education (DSME) of the University of Melbourne and three calculator companies, will conduct a three-year research project ***Informing policy change in mathematics curriculum and assessment: The challenge of computer algebra systems***. This project will investigate possible changes to mathematics curricula, assessment and teaching in Years 10 to 12 that would arise from widespread availability and use of CAS by students and teachers in the future. In particular, the project will consider the need for appropriate curricula, for CAS active VCE mathematics subjects, and will plan how these could be assessed. This project is funded by a major grant over three years from the Commonwealth Australian Research Council (ARC) Strategic Partnership with Industry Research and Training (SPIRT) scheme with support from industry partners Casio, Hewlett-Packard and Texas Instruments. Access to CAS would impact significantly on many current types of questions, especially those that involve the straightforward application of mathematical skills. A skills based CAS active tasks could include questions such as the following:

Part 1

Draw the graph of the cubic function with the rule $c_1(x) = x^3 - 3x^2 - 24x + 80$ and clearly identify all key features.

Part 2

Show that the cubic function with the rule $c_2(x) = -8x^3 + 36x^2 - 54x + 27$ has a single stationary point of inflection.

Part 3

Find the rule of a cubic function that has no stationary points and verify this. Obtain a general form for this type of function.

Part 4

Consider the following conjecture: "For any cubic function, c, with rule $y = c(x)$, there is a real number k, such that the points of intersection of $y = c(x)$ and $y = k$ are symmetrically located with respect to some point on the line $y = k$."

a. Select a cubic function, with turning points, that has only one x axis intercept, and determine whether the conjecture is true for this function for some value of $k \neq 0$.

b. Investigate whether the conjecture is true of cubic functions more generally.

More extended 'analysis' tasks provides teachers and students with the opportunity to engage in context based exploration of a topic or mathematical idea in some depth. CAS can be especially useful in supporting student work on analysis tasks where:

- collections of results need to be obtained efficiently, such as tables of values or families of graphs
- different examples of a given 'type' need to be developed fairly readily, such as graphs of a family of functions and their derivatives for specific combinations of defining properties or parameters
- involved arithmetic computation or algebraic manipulation needs to be carried out quickly and accurately, such as the solution of equations requiring numerical methods, systems of equations involving multiple variables, complicated anti-derivatives and numerical integration
- the focus is on the analysis of results which have been previously obtained, rather than production of those results
- results obtained by hand need to be checked for accuracy or reasonableness

Such an analysis task may involve consideration of rates of flow of liquid into various shaped containers, and involve both analytical and numerical approaches to the differential equations involved. Alternatively, when a curve is described by the rule of a function over a specified interval, the Pythagorean relation for finding the length of a line segment or a sum of line segments can be adapted to define a definite integral which can be evaluated either numerically or analytically to determine the length of corresponding arc of the curve. The use of arc lengths as a context provides students with the opportunity to see another application of the definite integral in a measurement context, and to apply numerical and analytical techniques to the evaluation of definite integrals. An analysis task could be developed using three components:

– use of the Pythagorean relation to evaluate the length of a line segment defined by considering the relation $y = mx + c$ over an arbitrary interval $[a, b]$
– consideration of a simple curve such as $y = kx^2 + c$ and informal use of a limit process involving the Pythagorean relation applied to a subdivision of the curve into straight line segments over an arbitrary interval $[a, b]$ followed by formalisation of this process to define the arc length of a curve defined by a function f over the interval [a, b] by:

$$S_f(a,b) = \lim_{dx_i \to 0} \sum_a^b \sqrt{dx_i^2 + dy_i^2}$$

$$\Rightarrow S_f(a,b) = \lim_{dx_i \to 0} \sum_a^b \sqrt{\frac{dx_i^2}{dx_i^2} + \frac{dy_i^2}{dx_i^2}} dx_i$$

$$\Rightarrow S_f(a,b) = \lim_{dx_i \to 0} \sum_a^b \sqrt{1 + \left(\frac{dy_i}{dx_i}\right)^2} dx_i$$

$$\Rightarrow S_f(a,b) = \int_a^b \sqrt{1 + f'(x)^2}\, dx$$

where the δx_i are subdivisions of interval $[a, b]$ over which the arc length is to be found

– application of these ideas to find exact or approximating functions for $S_f(a, b)$ for various functions f. These applications could be based in contexts such as the track of a 'Mad Mouse' ride, or the motion of particles described by cartesian or parametric equations, for example, the length of travel of a particle under projectile motion.

CAS can be used to develop tables of values for numerical approximations to arc lengths over different intervals, explore limiting values of arc length sum expressions, graph functions and evaluate definite integrals numerically and analytically. While the notion of arc length is fairly intuitive, as is the 'sum of small line segments' estimation approach, the evaluation of exact lengths requires some complicated integration techniques or numerical integration for relatively simple functions f.

To explore arc lengths, or approximations to these lengths, for different functions, CAS could be used to define the function *f*, define the definite integral corresponding to the required arc length (or approximation to this arc length) and evaluate this definite integral:

$$f[x\_] := \text{the rule of the function in terms of } x$$

$$S[f\_, a\_, b\_] := \int_a^b \sqrt{1 + f'[x]^2}\, dx$$

$$S[f, a, b] \text{ will evaluate the arc length from } a \text{ to } b \text{ of the function } f$$

For example, if we consider the function defined by $f(x) = mx + c$:

$$S[f, a, b] = \sqrt{1 + m^2}\,(b - a)$$

For the function with the rule, $f(x) = \log_e x$, the corresponding result is:

$$-\frac{\sqrt{1+\frac{1}{a^2}}\; a\left(\sqrt{1+a^2}+\text{Log}[a]-\text{Log}\left[1+\sqrt{1+a^2}\,\right]\right)}{\sqrt{1+a^2}}+\frac{\sqrt{1+\frac{1}{b^2}}\; b\left(\sqrt{1+b^2}+\text{Log}[b]-\text{Log}\left[1+\sqrt{1+b^2}\,\right]\right)}{\sqrt{1+b^2}}$$

Students should also be able to provide geometric interpretations for some of these results. While some functions such as linear functions, simple quadratic functions and log functions have accessible analytic forms for $S_f(a, b)$ others will require the use of numerical integration over specified intervals. For example, when $f(x) = \sin x$, the following shows the analytical and numerical results for $S_f(0, 2\pi)$, as well as a line segment approximation over four sub-divisions of length $\pi/2$, correct to 5 significant figures :

$$\int_0^{2p} \sqrt{1 + (\cos x)^2}\, dx = 4\sqrt{2}\,\text{EllipcE}[\frac{1}{2}]$$

$$\text{NIntegrate}[\sqrt{1 + (\cos x)^2}, \{x, 0, 2p\}] = 7.6404$$

$$N[4\left(\sqrt{1 + \frac{p^2}{4}}\right), 5] = 7.4484$$

In some cases the '1' under the square root term in $S_f(a, b)$ can be 'ignored', and a reasonable approximation to the exact value obtained. Students could consider functions and intervals for which this may be the case.

The CAS-CAT research project

Systemic education curriculum and assessment authorities, internationally and nationally, have, over the past decade, grappled with policy issues related to graphics calculators (Stacey, Dowsey, McCrae and Stephens, 1998). CAS is the context for the next range of issues to be dealt with, however the potential effect of CAS is dramatic because of its broad application across mathematics and because it directly involves the secondary-tertiary interface. The role of CAS at this interface will be critical to future direction in the use of such technology in mathematics education (Leigh-Lancaster, 1996). As the availability of CAS in schools increases, so does the need for the official curriculum to produce appropriate policy responses. Effective policy development on the possible use of CAS in formal examination-based assessment requires rigorous and well-grounded research, especially because this is associated with the highly sensitive credential requirements for tertiary study.

The research project aims to investigate the changes that regular access to CAS super-calculators will have on senior mathematics subjects and the associated assessment in Victoria, Australia and to explore the feasibility of offering new mathematics subjects which use CAS extensively.

The Chief Investigators of the project are Gary Asp, Helen Chick, Barry McCrae and Kaye Stacey from the University of Melbourne, and David Leigh-Lancaster from the Victorian Board of Studies is a partner investigator. The four industry partners are the Board, and three calculator suppliers and manufacturers: Hewlett-Packard, Shriro (Casio) and Texas Instruments. The industry partners will supply CAS super-calculators to students in three schools for a three year program of classroom based research. With the cooperation of the Board, the content and formal assessment undertaken by these students for Year 12 will be altered, culminating with the trial in volunteer schools in 2002 of a VCE examination base on a CAS active mathematics subject, instead of the current Mathematical Methods subject. The time line envisaged for the stages of the project is summarised in *Figure 2* below. Actual implementation of this requires the Board's continuing approval, which will be informed by the findings of the project in earlier years. The outcome will be substantial policy advice to the Board, practical feedback to the calculator manufacturers, and important research insights into questions of learning mathematics in a technology-rich environment.

For each project school CAS support from one industry partner		
2000	**2001**	**2002**
One Year 10 Maths class ➔	➔ Year 11 Maths Methods ➔	➔ Year 12 New CAS Maths Assessment: New Board CAS paper
One Year 11 Maths Methods class ➔	➔ Year 12 Maths Methods Assessment: Standard VCE + trial CAS paper	
	One Year 11 Maths Methods class ➔	➔ Year 12 New CAS Maths Assessment: New Board CAS paper

Figure 2 - time-line of research and assessment changes for each project school

For the calculator industry partners, the project enables their products to be tested seriously within an Australian curriculum context, investigating the suitability of the products' capabilities, interfaces, notations and physical characteristics. Materials for training teachers to use the CAS super-calculators will be enriched by experiences in the project schools. For the Board, the project will result in advice to support the development of policy for curriculum and assessment, covering issues such as:

- which parts of Year 12 assessment could permit CAS, forbid it or require it;
- the protocols for the use of CAS super-calculators in assessment;
- how (and if) examination questions can be set to be fair to users of CAS super-calculators of different brands and models, given that they have different capabilities (the involvement of different industry partners is critical for this);
- subsequent redevelopment of lower secondary mathematics curricula and suggestions for post-secondary mathematics and mathematics-related studies.

Beyond these practical issues, the scientific importance of the study lies in its advancement of our detailed knowledge of how students learn mathematics and how it can best be taught. This new learning environment provides us with new opportunities for research into the teaching of mathematics. When students have CAS in class and in examinations, the need for memorising routine procedures may be enormously reduced, yet the need for conceptual and structural understanding is almost certainly undiminished and possibly enlarged. Effective policy development (in particular for assessment used for university selection and pre-requisites) requires rigorous and imaginative research, especially as the possible changes are likely to be substantial and subject to robust debate. We know of no similar study being undertaken anywhere in the world. Thus, the results of this research are likely to have implications for education systems throughout Australia and, indeed, the rest of the world.

In the rest of this paper, we present our preliminary thinking on one of the major policy issues to be resolved through the project: the way in which assessment will use or not use CAS. We recognise that this is likely to be a sensitive and possibly political issue and so careful detailing of arguments and analysis of data will be needed to guide policy.

Systemic adoption of CAS

Examining mathematics with access to CAS will present more challenges than examining mathematics with graphics calculators. McCrae (1996) found that access to graphics calculators would impact on only 6% of a 1994 VCE Specialist Mathematics paper (the hardest mathematics subject in Victorian schools), but that about 60% would be affected by the availability of CAS. Similarly, Shumway (1989) reported that about 90% of the exercises in most US textbooks could be computed directly by CAS. The adoption of CAS in teaching is therefore inherently associated with the adoption of CAS in assessment.

As yet, only a few countries around the world have national policies permitting the use of CAS in examinations. In Denmark, CAS will soon be permitted in all mathematics examinations for 15-19 year old students. In France, any calculator, including those with CAS, is permitted in examinations. In parts of Germany, teachers can decide whether CAS is permitted in lessons and examinations. Elsewhere around the world, some countries have now permitted graphics calculators but not CAS (as in Victoria), some (such as Italy and Ireland) are about to introduce pilot projects, and some, often in Asia but also including poorer nations, do not use calculators at all. The unique nature of each country's assessment regime and the national priorities makes research in individual countries important.

One policy likely to be influential in Australia is that of the USA College Board for Advanced Placement (AP) Calculus (USA College Board, 1999). The long list of calculators permitted includes both graphics and CAS super-calculators, although those with QWERTY keyboards, and pocket organisers and pen-input computers, are not allowed. From 2000, both multiple-choice and free-response sections will be in two parts: one where some parts require the use of a graphics calculator and one where the use of any calculator is not permitted. The web-site notes that: "This change in format is an effort to respond to heightened concerns with equity as more students may use graphing calculators with computer algebra system (CAS) features." It is claimed that the two part format will provide flexibility in the types of questions that can be asked and also ensure greater fairness to students, regardless of calculator used. In addition, it is specified that students can only use a calculator for three operations: solving an equation, finding a derivative, or calculating the value of a definite integral. In these cases, the student must indicate the set-up of the problem (eg., write down the relevant definite integral for finding an area before evaluating it by calculator). In all other cases, the student must show the "mathematical steps necessary to produce the results". For example, to find a local minimum, normal calculus procedures must be followed to establish the derivative and set it to zero. Many of the capabilities of CAS therefore cannot be used.

Technology-free, technology-neutral or technology-active?

Stephens and Leigh-Lancaster (1997) write that there are three possible positions that should be investigated for the use of CAS technology in examinations:

- that assessment (of at least some areas of mathematics) should be *technology-free*, ie., that students should not use technology (or at least advanced or "new" technology) in the assessment;
- that assessment should be *technology-active*, ie., that students should be permitted to use specified advanced or 'new' technology in assessment and that some questions should be designed to require its use; and
- that assessment should be *technology-neutral*, ie., that students not using technology in an assessment should be able to answer questions as easily as those using technology.

Victorian mathematics subjects have in recent years passed from technology-free (with regard to *graphics* calculators, although scientific calculators have long been permitted), to technology-neutral, to technology-active external assessment. This experience indicates that the technology-neutral position is certainly not sustainable in the long-term, as it is very hard to set questions which are genuinely technology-neutral (McCrae, 1996).

Kemp, Kissane and Bradley (1996), note that in regard to graphics calculators:

> "… the use of calculator neutral examinations is an unwise long-term strategy, although it may be seen as helpful in the short term to allay concerns about disparities in student access to graphics calculators. In the long term, such a strategy would send a clear (and incorrect) signal that graphics calculators are not of importance in mathematics, and would discourage both students and their teachers from acquiring either hardware or expertise in its use."

In designing its examinations, AP Calculus has combined the three assessment positions stated above. Technology-free assessment has been chosen for part of the assessment and the other part is technology-active assessment, partially neutralised. It is technology-active in the sense that a CAS calculator is permitted and some parts of some questions require its use. On the other hand, the use is constrained to three operations. The College Board has given two reasons for this choice (http://www.collegeboard.org/ap/calculus/html/exam002.html): firstly, the recent introduction of graphics calculators has placed a large burden on teachers, who need time to adjust their courses. Secondly, the College Board notes that it can develop fair examinations with any specified type of technology, but it cannot develop exams that are fair to all students if the spread of capabilities of the technology is too wide. The College Board has endorsed the use of any technology and promoted it in teaching — accepting that some students will have CAS whilst others have basic graphics calculators — but constrained their use in assessment in order to increase equity.

An initial position (which may be modified in the light of experience and data) is that the CAS-CAT project should aim for technology-active assessment only. The reasons for this initial decision are explained in the next section. However, it is expected that this position will be feasible provided only a narrow range of (high) technology capabilities is permitted and if teachers are provided with adequate professional development before they begin to teach the new subjects. With time, the capability of these machines will increase, as more advanced technology becomes affordable. Furthermore, if the new subjects are initially phased in as parallel subjects to the current subjects, teachers can decide to move their classes across to the new syllabus when they are themselves professionally ready and when typical students in their school community can afford to purchase an adequate machine. The potential for serious equity considerations arising from the cost of such new technology is evident. It is unlikely that these will be resolved, in the foreseeable future, however the development of appropriate policies may be able to reduce the impact of these concerns.

Although the project does not intend to investigate technology-neutral assessment (for reasons given above), one of the features of the research design is that it will allow a full exploration of the practicality of *brand-neutral* assessment. As hand-held machines become more and more powerful, the divergence between the capabilities of different models may either increase markedly, or alternatively decrease with "product maturity". Current experience with graphics calculators shows that questions that are challenging on one machine can be straightforward to deal with on another machine. For example, an important point of an assessment item may be to test students' understanding of the shape of a graph. Sometimes this can be done by using a graph that does not appear in a standard viewing window, so that the student needs to know properties of the graph to locate a desired feature. However, facilities such as automatically locating zeroes and turning points are available on some calculators, making some such questions straightforward. The capabilities of different models of CAS active super-calculators will need to be fully appreciated in order to set brand-neutral assessment items.

Why plan only technology-active assessment?

Congruence plays an important role in adopting an initial position to create technology-active, rather than technology-free or technology-neutral assessment. This does not mean that every question will require use of technology (although some questions will clearly be devised to require such use), however technology would be available for use on any question. A key reason for this initial position relates to values and beliefs about mathematics: mathematics at school should be like mathematics as used outside school. As mathematics outside school changes and the methods of choice change, so also should the methods of choice at school change, to the extent that this is reasonably possible. National associations have endorsed the principle that assessment should be aligned with teaching and learning as closely as possible for many years (see, for example, AAMT/CDC, 1987). It may well be the case that a technology-free component of assessment would endanger mathematics remaining a sensible subject where students learn to use up-to-date methods.

Many believe that the decisions to make senior school mathematics arithmetic-calculator-active, and more recently graphics calculator active, have been sound decisions, and that this will also be the case for algebra. A final argument for technology-active assessment only is that within time limited assessment episodes, it maximises the time that is spent on assessing higher-order thinking skills, rather than routine procedures.

What are the arguments for a technology-free component of assessment? One is the need to provide fair assessment for students with different types, models and brands of calculators. An approach to minimising this concern has been outlined earlier. The research project will provide some data to judge this approach. A second argument is that tertiary mathematics courses may not use CAS and hence students must not rely on it. This may not be the case for much longer than the time frame of the proposed curriculum change. A third argument is that the existence of technology-free assessment would encourage the acquisition of important mental or by-hand skills and a fourth argument is that technology-free assessment is better able to test ‘true understanding’ — what students really know and really understand. These two final arguments are substantial, and reach to the heart of values and beliefs about mathematical activity.

How can mental and by-hand skills be encouraged?

It is taken as axiomatic that an important function of an external examination system is to encourage good learning and teaching. There are skills, especially algebraic skills over which students should have personal mastery and the existence of technology-free assessment would encourage the acquisition of these important mental or by-hand skills. Years of teaching with scientific calculators have shown that to achieve the goal of sensible and powerful mathematics, students cannot be only taught how to carry out arithmetic on a machine. They must develop a strong number sense, one that enables them to operate quickly and effectively in the world of informal arithmetic (eg., “Am I being charged about the right amount here?”) and also to operate a machine competently, guarding against errors by effortlessly monitoring the results of calculations.

At the same time, there has been a re-assessment of what students should know, and in our time, by-hand procedures such as taking square roots and division using log tables have been consigned to history. Similar considerations will also apply with CAS — students will need well-developed symbol sense (Arcarvi, 1994) and a mastery of some simple algebraic procedures, while other currently taught procedures may no longer be required. The capacity of students to develop knowledge and understanding of important algebraic concepts and skills in a CAS active environment, and how to best monitor this development, is a key question for researchers in this field. If algebraic insight and symbol sense is made a goal of the assessment, it is not a priori unreasonable to expect that it can be assessed as well with calculators as without by clever question design. On the other hand mastery of simple algebraic procedures is absolutely essential for smooth and effective CAS use - students who do not have this mastery are unlikely to do well on a technology-active examination. This will be implicitly tested, rather than explicitly. It will therefore be important to be quite explicit about these issues in advice to teachers.

This argument assumes that only simple algebraic procedures will be required to be able to be done 'by-hand' as a mathematics professional, or mathematics user, in the future. Exactly what constitutes a simple procedure will vary from person to person, as it does now with arithmetic. This however, is likely be a vigorous area of debate for many years. Kutzler (1999) in proposing the idea of assessment of “intellectual fitness” is beginning this debate.

Does true understanding depend on being able to carry out the steps?

The fourth argument for technology-free assessment is that it is better able to test “true understanding” than is technology-active assessment. In fact, it is tempting to believe that what a person “really understands” is what he or she can do from memory or “by-hand”. Pierce (1999) has shown how students themselves often believe this. Unfortunately, however, dissatisfied mathematics students across generations have attested to the fact that they did not understand procedures that they learned to carry out by hand successfully. Technology-free assessment therefore cannot necessarily claim to reliably identify ‘true understanding’: the question is whether technology-active assessment can identify it and encourage it any better. Experience in assessing with graphics calculators leads us to expect that this may be able to be achieved with careful question design.

The question of what constitutes 'understanding' (with possibly different answers for different types of users of mathematics) is a much deeper question, one that is likely to be always be open to further research and hopefully, insightful reflection.

Year 12 mathematics is critical to prepare Australian young people for a technological future. Mathematics subjects are the principal pre-requisites for tertiary study in the sciences, engineering and economics, on which Australia's economic competitiveness depends. It is clearly important that our students learn to use the mathematical tools of the future, but also that a cautious approach be adopted so that a sound balance between traditional by-hand algebra skills and technology use can be achieved.

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