# Renewal of Educational Senses

# Creation of Technical Conditions

# Reform of Mathematics Educations

# Adaptation to Development of the Times

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**Abstract** This article discusses the core issue of the ongoing reformation of mathematics education all over the world, that mathematics education must keep up with the rapid development of science and technologies and the trends of the age. It proposed a goal that we should achieve: renewing our idea of education,creating technical conditions, reforming mathmatics education to make it meet the demands of the development of the current hi-technology society and the information age, and training talented people with creativity and capability in sustainable development. An effective way to achieve the goal is also given here --- computer- and calculator-assisted instruction.

**Keywords** mathematics education computer- and calculator-assisted instruction

## Our goals

We are now in the 21st century. From the exploration of the universe to the cloning of creatures, from the information technology to the Internet, the world is changing and developing all the time, and all kinds of high technologies are emerging in endlessly.

Mathematics education must keep up with the rapid development of science and technology and the trends of the new age. This idea is shared by all the people working on mathematics education, and is also the core issue of the ongoing reformation of mathematics education.

Mathematics education should not only stay in the traditional form of 'teacher speaks, students listen' and the strict deductive system of formal logic from axioms and definitions to theorems and correlations; it should be open, should keep its close relation to its practical background and social environment, and should strengthen its link with other related subjects. This is the challenge to the mathematics education nowadays.

Therefore, our goal can be described like this:

Renew the idea of education, create technical conditions, reform the mathematics education to make it meet the demands of the hi-technology society and the information age, and train talented people with creativity and capability in sustainable development.

In order to achieve this goal, we should first update our guidelines and conceptions. There are a lot of problems to think of:

What is the goal of mathematics education?

What is the principle of mathematics education?

What is the relationship between mathematics education and the real world?

What is the relationship between mathematics education and the development of science and technology?

What should be taught in mathematics?

And how should we learn mathematics?

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It is important that the answers to these problems should change with the background of age and the society.

According to the constructive cognition theory, learning mathematics is a continuous process of assimilating new knowledge and constructing new meanings. It will be effective only if the students do the learning activities actively by themselves, and will be successful only if the students experience it themselves and build up their confidence.

In *Guidelines on Mathematics Education in the 21st Century for Middle and Primary Schools*, proposed in Shanghai, it was maintained that the reformation of mathematics education in the 21st century should be based on the development of student. This just reflects the fundamental requirement on school education in a modern society, the most important rule in education, as well as the new characteristics of modern education, such as activeness, democracy, cooperativeness and multiformity.

The development of students greatly depends on the formation of the students' self sense and activeness. It is crucial that the students learn to gain mathematical knowledge by themselves, to participate in mathematical practices forwardly, so that they will gain the basic mathematical ability and creativity which will be useful throughout their lives.

When carrying out quality education, we should see not only the students' differences determined by their natural individuality, but also their commonness determined by their sociality. According to the theory of social constructivism, the individuals must reside in a social environment, participate in the related practical activities actively, and construct knowledge forwardly through the communication and cooperation between individuals and the collectivity.

The quality of a talented person consists of two levels, subsistence and development. The subsistence level mainly includes the physical and mental basis, and needs of basic living, self safety and labor, i.e., basic qualities for survival. The development level includes ideologic, scientific, technologic, physical, mental and specialized qualities, which are the qualities required for realizing higher desires beyond basic survival.

Therefore, how to reform the traditional ways of classroom instruction, to create a environment for the students to improve their interpersonal communication using modern technologies, will become a important precondition for carrying out the quality education based on the development of students.

Secondly, we must get prepared on the teaching environment, technologies and tools. A main difference between human and animals is the ability of human to use tools. In a mathematical sense, from the calculating sticks and abacus in the ancient times, to the recent computers and calculators, all these have become useful tools for mathematics learning, and great helpers for mathematics teaching.

It is commonly accepted that "the basis of high technology is applied science, and the basis of applied science is mathematics"; there are also sayings like "the essence of high technology is a mathematical technology". The scope of mathematics has expanded to the data and results of measurements and observation in science, mathematical deduction and proof, mathematical models for natural phenomenons, human behaviors and social systems.

Mathematics is regulating and constructing the possible forms of the real world; at the same time, calculation technologies and universal concepts process these mathematical patterns in the world. They have become the two crucial characteristics of the mathematical development in the 20th century.

In the upcoming 21st century, economic grow based on knowledge and information will become a more and more important way of economic development. The ability to create and use knowledge and information will become the core of the competibility of a country. The quality and ability of the people will become the precondition of intellectual economy.

The rapid development of science and technology, especially the coming of the information age, demands more mathematical ability on the people. Modern high technologies are gradually showing up as mathematical technologies. The development and application of high technology are making use of modern mathematics in people's everyday life in a technical way.

"Digitalized economy", "Digital information processing", …, these new popular terms, and the close relationship between computing technologies and the analysis, observation, experiment and simulation of a large amount of explorative data, greatly shows the two-sidedness of mathematics, being both science and technology. Modern technology has become the essencial meaning of mathematics, an important side to reflect the features of mathematics, and has given mathematics the characteristics of a experimental science. Modern technology has been implied in mathematics.

The recent development of mathematics shows that mathematics is no longer only a strict theoretical system, a sea of pure logical deduction. Mathematics is an open system, is a subsystem of the human cultures. The external environment has provided the development of mathematics with great motivity and adjustment, while modern technology acts as an important link between mathematics and the real world, is the embodiment of the great motivity.

According to the ways scientific knowledge forms and develops, we can divide mathematical activities into three aspects:

1) Accumulation of facts by observing, experimenting, inducing, comparing, and generalizing.

2) Abstraction of the facts into raw concepts and axiom systems, and construction of theories based on these concepts and axiom systems.

3) Practical application of the theories.

Meanwhile, mathematics instruction should be the process in which the students participate in mathematical activities led appropriately by the teachers. From that, it is easy to realize that learning mathematics is also a specific kind of mathematical activity.

After learning mathematics, in order for the students to learn not only the basic knowledge and theories but also the respective abilities, not only basic abilities but also abilities of development, even innovation, our mathematics education must give the students more freedom to try out, and a active environment for development.

Mathematics education must strengthen its relationship between the real world and other subjects, must give the students the opportunity to explore the underlying rules, create models, re-create the problem and find its mathematical patterns, in a real background and the corresponding micro world, and ultimately reach the goal of solving the problem.

In order for the students to mathematicalization and recreatation the problem successfully, we should rely on the teaching environment created for us by modern technologies, in which students learn mathematics as if they are in a "mathematical lab", they can observe and try out errors, can do discoveries and make conjectures, can do experiments by measuring and categorizing, can design algorithms and check the conjectures with calculation, and can also propose prepositions and prove it or negate it using logic reasoning, and so on.

To sum up, renew the ideas, reform mathematics education, make it meet the demands of the age, the central thing to do now is to create a teaching environment with modern technologies, and with the active participation of the student, the education will be based on the development of the student, and our goal will be achieved.

## An effective way to achieve the goal --- computer- and calculator-assisted instruction

Our goal is big and the task is heavy. We found that computer- and calculator-assisted instruction is an effective way to achieve the goal and fulfill the task.

The development of modern science and information technology creates a large variety of ways of teaching mathematics. Calculators, computers, multimedia technology, slides, videos, records, and even animations, network, … Each technology has its features, and can create non-standard new teaching environments in all kinds of situations, and consequently, achive our goal of reforming mathematics education to make it meet the demands of the information age.

Computer-assisted instruction is now familiar to many mathematical educators, and has become a effective way to achieve such a goal.

The rapid development of graphing calculators recently, like the great breakthrough on functionality of products such as TI-92, TI-83 (we can call them "hand-held computers"), has also given us great convenience. These graphing calculators include many powerful softwares, such as a symbolic algebra system capable of doing symbolic algebraic calculation, integration and so on, an interactive geometry system capable of making geometric constructs and transforming the constructs dynamically for exploration, and a data analysis system capable for exploring the rule of the data, doing regression analysis. It also supports programming.

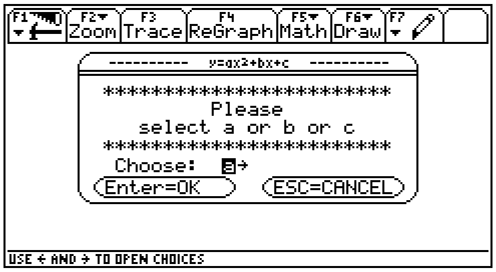
Based on the TI graphing calculators, there are devices like "Calculator-based Laboratory" (CBL) and "Calculator-based Ranger"(CBR) which can collect and process physical and chemical data from the real world, like distance, temperature, sound, light, force, etc., and furthermore, analyze them using mathematical techniques.

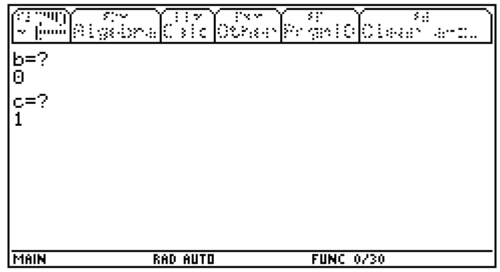
A graphing calculator can transmit data, graphics and programs to and from a computer or another calculator, thus makes it easier to share, modify, save and output them. The new Flash technology allows downloading newly-developed programs from Internet and updating the softwares in the calculator, making it more useful.

Being an effective way to achieve our goal, computer- and calculator-assisted instruction has the following unique advantages:

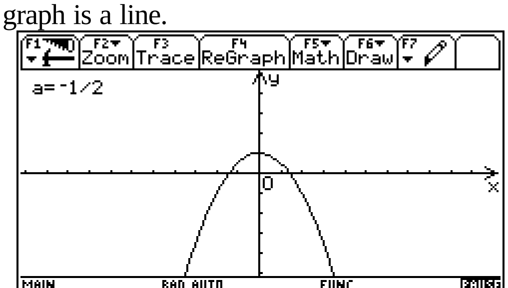
1) The instruction based on the development of students can be well embodial. Students participate in actively, explore by themselves, gain knowledge by their own eyes, thus lead to a sustainable development of ability.

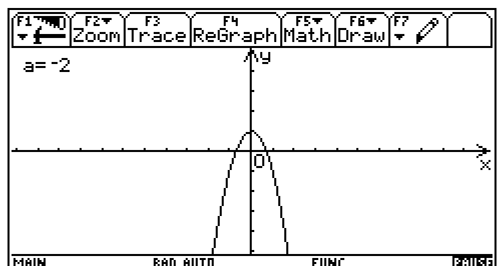
**Example 1** In the quadratic function $y = ax^2 + bx + c$, observe the effects on the graph of changing the coefficients *a*, *b* and *c*.

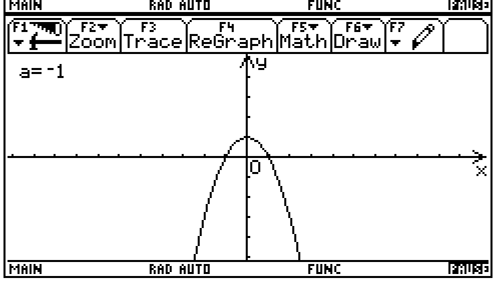


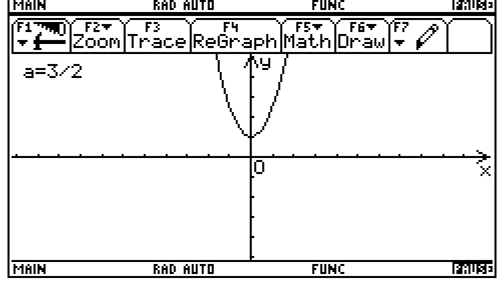




If we want to see the effects of changing *a*, we should make *b* and *c* constants. For example, we assume *b*=0, *c*=1, then use an animation to see how the graph changes with *a* going from –2 to 2 (step 0.5). From the animation, we know that the value of *a* decides the direction and size of the parabola's opening. When *a* is negative, the parabola opens downwards. When *a* is positive, the parabola opens upwards. We can also see that when *a*=0, the function is not a quadratic one, and its graph is a line.

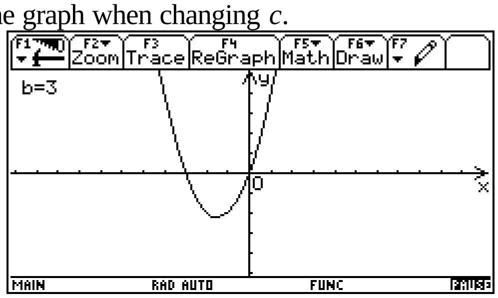


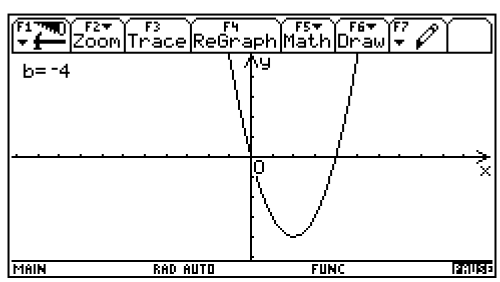



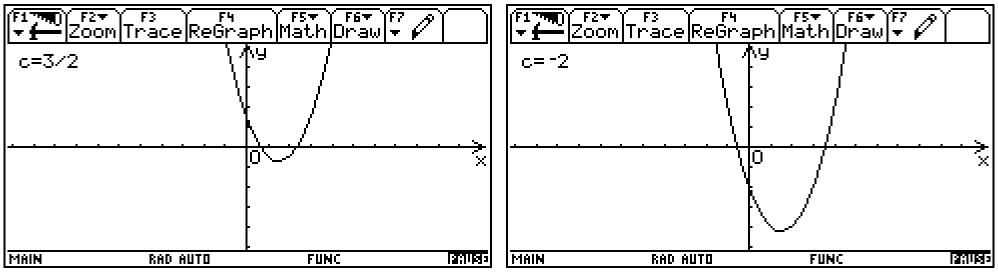




Using the same method, we can see the effects on the graph of changing *b* or *c*. For example, let *a*=1, *c*=0, observe the change of the graph when changing *b*; let *a*=1, *b*=3, observe the change of the graph when changing *c*.





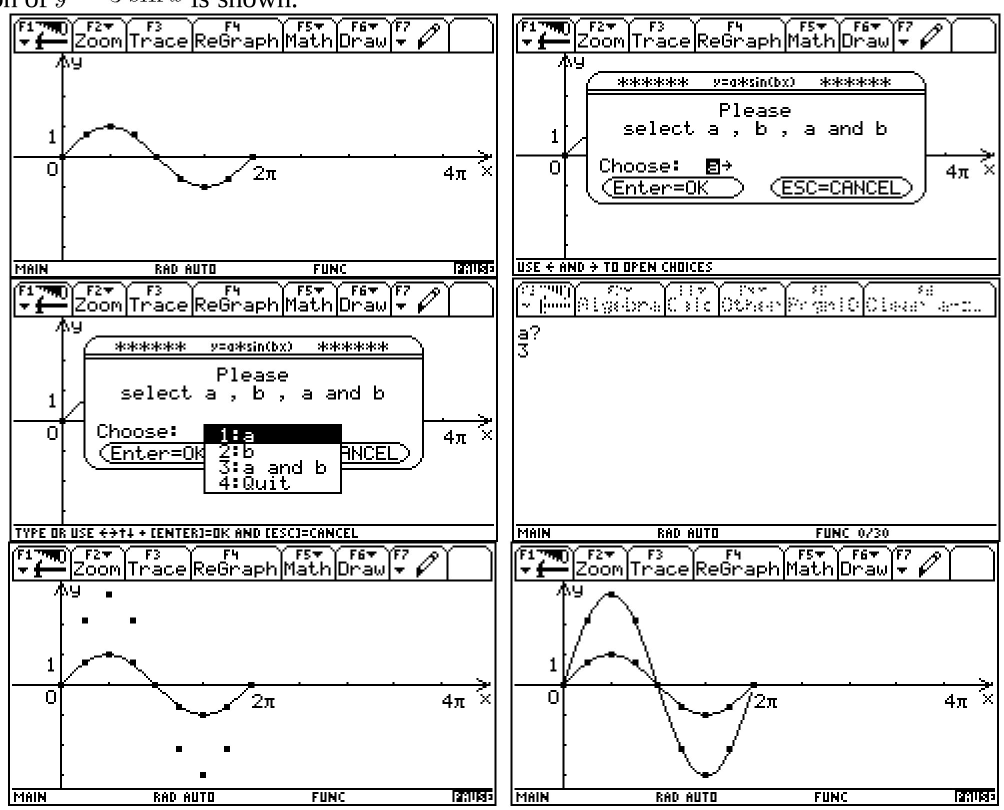

By participating in the activity, exploring and concluding the rules by themselves, the student will learn the relationship between the coefficients of a quadratic function and the features of its graph by heart, and will set up a firm basis for further exploring the characteristics of quadratic functions.

**Example 2** In the function $y = a \cdot \sin(bx)$, observe what *a* and *b* does.

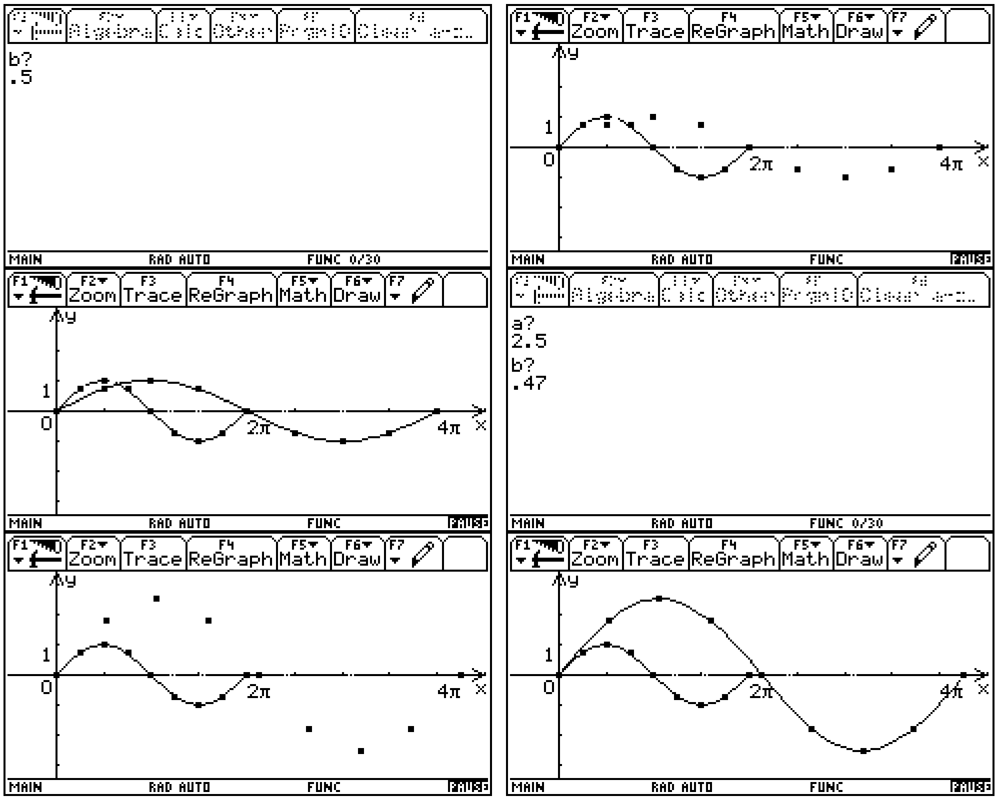
First, we run the program, draw the graph of $y = \sin x (0 \leqslant x \leqslant 2\pi)$, and select the following 9 points on the graph:

$$(0,0), \left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right), \left(\frac{\pi}{2}, 1\right), \left(\frac{3\pi}{4}, \frac{\sqrt{2}}{2}\right), (\pi, 0), \left(\frac{5\pi}{4}, -\frac{\sqrt{2}}{2}\right), \left(\frac{3\pi}{2}, -1\right), \left(\frac{7\pi}{4}, -\frac{\sqrt{2}}{2}\right), (2\pi, 0)$$

Then, choose the coefficient to change (*a*, *b*, *a* and *b*) from the dialog box. For example, change *a* to 3, we can see the 9 points move vertically to be three times as far to the x axis, and the graph of $y = 3\sin x$ is shown.




If we change *b* to 0.5, the 9 points will move horizontally respectively, and the graph of $y = \sin(0.5x)$ will be shown. If we change *a* to 2.5, *b* to 0.47, the 9 points will move vertically, then horizontially, and the graph of $y = 2.5\sin(0.47x)$ will be shown.




By having the students participate in the activity positively, the atmosphere in the classroom becomes better, and the students' understanding of graph of the sine functions are deepen, especially the effect of *a* and *b* on the amplitude and period of the graph. The students' active position is fairly well embodied in this exercise.

2) The fact "the teaching of mathematics is the teaching of mathematical activities" can be realized. With devices like computers and graphing calculators, the process of mathematics teaching will reflect the facts in micro world, like being placed on a reality background, and also create relationship between other subjects spontaneously. Thus, the students will see the process of the formation of mathematical knowledge, the creation of mathematical models, and the exploration of the underlying rules, by their own eyes.

**Example 3** Assume a travelling company's annual profit is as follows:

| *Order* | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Year *x* | 1987 | 1988 | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 |
| Profit *y* (in $\$10^4$) | 7 | 20 | 31 | 38 | 45 | 47 | 51 | 53 |

Create an appropiate model between *x* (number of years from 1987) and *y* (annual profit in $\$10^4$), calculate its parameters, and predict the company's annual profit in the year 2000.

According to the students' observation of the scatter plot, three models are proposed:

(1) Logarithmic model $y = a \cdot \ln x + b$, where the parameters *a* and *b* are calculated using a computer or a calculator: *a*=22.9894, *b*=6.0258. Thus $y = 22.9894 \cdot \ln x + 6.0258$, the correlation coefficient $r$=0.9974, $r^2$=0.9948, the predicted annual profit in 2000 is $666,962.

(2) Arccotangent model $y = a \cdot arc\cot x + b$, where $a = -70.5311, b = 58.0523$. Thus $y = -70.5311 \cdot arc\cot x + 58.0523$, $r$=0.9730, $r^2$=0.9468, the predicted annual profit in 2000 is $530,229.

(3) Quadratic model $x = ay^2 + by + c$, where $a = 0.0031, b = -0.0463, c = 1.3480$. Thus $x = 0.0031y^2 - 0.0463y + 1.3480$, $r^2$=0.9853, the predicted annual profit in 2000 is $717,919.

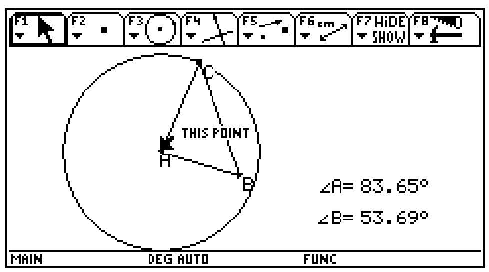
Among the three models proposed, model (1) best corresponds to the data.

In this example, based on their current knowledge, the students analyze the given data, create appropirate mathematical models, calculate the parameters in the models using methods like regression, predict the profit in the next years, and compare the models using some specific method. From that, they have learnt "real mathematics".

3) Have a thorough understanding of "the essence of modern mathematics". By using modern technical devices like computers and graphing calculators, in the process of exploring mathematical concepts, proving mathematical facts and solving mathematical problems, the students can use dynamic methods, and from dynamic and static ways, macroscopical and microcosmic viewpoints, especially from the tight relationship between mathematical facts and other subjects or backgrounds in reality, they can form a more comprehensive and correct view of mathematics.

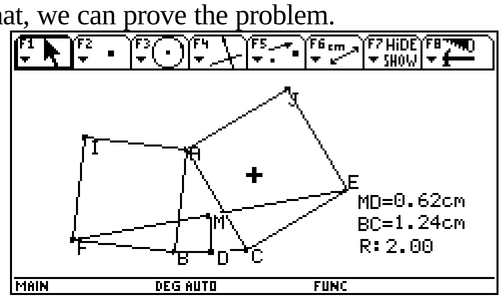
**Example 4** Assume two of a triangle's three sides have the length $a$ and $b$, respectively $(a \geqslant b)$. How long should the third side be so as to make the largest angle of the triangle the minimum?

With the geometry facility of the TI-92 calculator, we construct the triangle ABC, whose two fixed sides AC=$a$, AB=$b$. When dragging the moving point C on the screen, we observe that while C moves along circle A counterclockwise, angle B becomes smaller and smaller, angle A becomes larger and larger; conversely, while C moves clockwise along the circle, angle B becomes larger and larger, angle A becomes smaller and smaller. According to the measurements on the screen, we can see that when C is on the perpendicular bisector of segment AB, the largest angle of triangle ABC (A or B) is the minimum.




**Example 5** Assume B and C are two fixed points, A is a moving point to one side of BC. Draw squares ABFI, ACEJ outside of triangle ABC with one side being AB and AC, respectively. Prove that if A is kept to the same side of line BC, wherever A moves, the midpoint M of segment FE will not move.

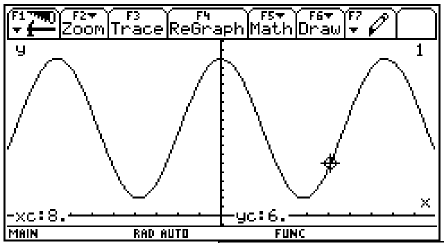
With the dynamic geometry facility of the TI-92 calculator, we make the following construct according to the problem. As we drag the moving point A on the screen, we can see that wherever A is (to the same side of BC), M is always on the perpendicular bisector of segment BC, and its distance to BC is fixed (half of the length of BC). At the same time, we can find the way to prove the problem when A is at some particular place, and the relationship between the particular case and the general cases. From that, we can prove the problem.

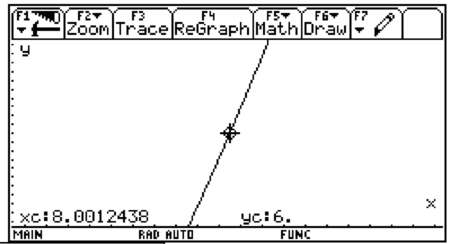


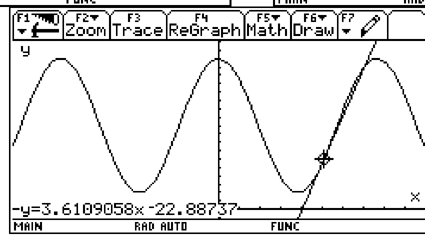

This dynamic way of thinking to solve mathematical problems, can be implemented using computers or calculators like what we have done above. This greatly shows the power of modern technology, and also shows that computer- and calculator-assisted instruction is an effective way to achieve our goal.

**Example 6** Assume $f(x) = 8 \cdot \cos(\frac{p}{6}x) + 10$, calculate its changing rate at the point $x$=8.

Derivatives reflect a function's local properties. Select `A:Tangent` from the F5 menu to draw the curve's tangent line at the point $x$=8. Its equation is $y = 3.6276x - 23.0208$.








Using the Zoom in facility to look closely at the curve near $x$=8. We can see that the curve looks very similar to the tangent line.

This shows that we can replace curves with tangent lines in local scope. With these observations, the students will have a better understanding of these mathematical concepts and their meanings.

The above examples shows the impacts of computer- and calculator-assisted instruction on mathematics teaching. These impacts are not reachable with traditional ways of mathematics instruction. We can believe that as we keep working hard, we will achieve our goal perfectly.

## Final words

In order to achieve the goal of the modernization of mathematics education, there exists many problems for further exploration:

How to combine modern technologies with mathematics education?

How to make use of graphing calculators in mathematics classroom?

How to assure the students' basic mathematical abilities?

How to balance using calculators and exercising the students' basic abilities?

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We hope all the mathematics teachers and educators will work and explore together, and find the best strategy and way to reform the mathematics education in China!